Visualization for Structured Constraint Satisfaction Problems

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Abstract

Constraint satisfaction problems are mathematical models of real-world problems. In contrast to randomly generated artificial problems, real-world problems usually have non-random structure. Knowledge about that structure, when identified in advance, can make search to find solutions more effective. This paper introduces DrawCSP, a visualization program that can show both the original and the discovered structure of constraint satisfaction problems. DrawCSP provides insight into both search algorithm design and into the challenges real-world problems present.

Introduction

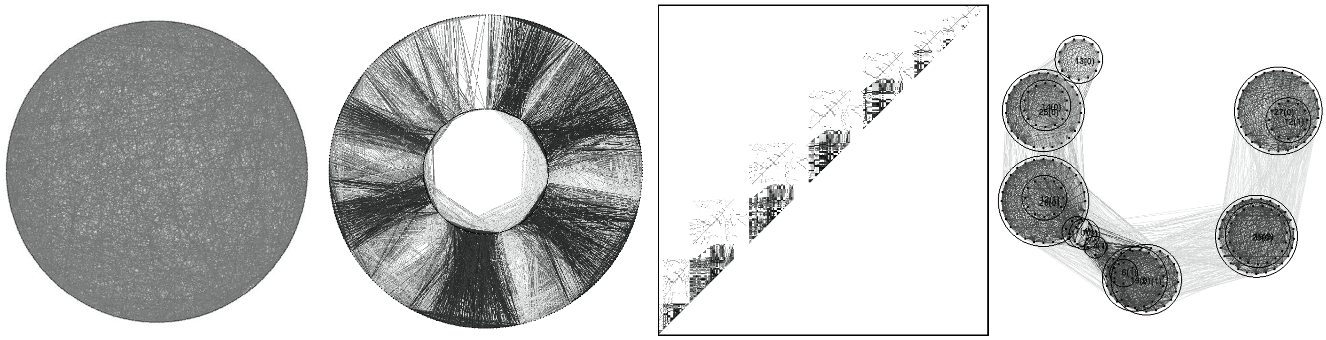
Constraint satisfaction is a paradigm that applies to many challenging real-world problems, including planning and scheduling. Traditionally, algorithms and heuristics to solve constraint satisfaction problems (*CSPs*) have drawn both inspiration and guidance from visualizations called *constraint graphs.* As solvers tackle more challenging problems, however, these visualizations become larger and more opaque. The thesis of this paper is that, because real-world problems have non-random structure, proper visualization can both support search algorithm design and provide insight into the challenges inherent in real-world problems. The principal results of this paper are several novel visualizations of structured CSPs, and the exploitation of that detected structure to improve search performance on challenging real-world problems.

Consider, for example, the driverlog problems from the Third International Planning Competition (Long and Fox, 2002). Each problem involves sets of drivers, trucks, locations, and packages. The goal is to deliver packages to different locations, and have the drivers and trucks finish at specified destinations. When cast as a CSP, a driverlog problem can be visualized as a constraint graph. The traditional constraint graph in Figure 1(a), for example, plots 650 points for one driverlog problem along the circumference of a circle, and includes 17447 lines, each of which joins a pair of vertices. This picture offers little information about the structure of the problem. Figures 1(b)-1(d) are products of the program DrawCSP. They offer more striking and more useful visualizations.

Formally, a CSP <*X*, *D, C*> is a set of variables *X*, a set of *domains D* (possible value sets for each of the variables), and a set of constraints *C.* Each *constraint* restricts the simultaneous value assignment of its *scope*, a subset of *X.* A constraint is *n-ary* if its scope is of size *n*. This paper deals only with *binary* CSPs, those with constraints whose scopes are at most of size 2. In the constraint graph *G* = <*V*, *E*> for a CSP <*X*, *D, C*>, *V* represents the variables *X*, and *E* represents the constraints *C*. Thus the driverlog problem in Figure 1(a) has 650 variables and 17447 constraints.

The *tightness t* of a constraint is the percentage of value tuples it excludes from the Cartesian product of the domains of its scope variables. In Figure 1 darker edges represent tighter constraints. (DrawCSP actually produces diagrams in color, where tighter edges are represented with deeper colors. The figures in this paper have been adjusted for maximum contrast in grayscale printing.)

The *density d* of a CSP is the fraction of possible constraints included. Although the density of the problem in Figure 1 is less than 0.1, its many constraints mask any structure from view, at least when the variables are arranged on a circle. To cope with this opacity, Figure 1(b) plots the same vertices on two concentric circles, odd-numbered vertices on the outer circle and even-numbered vertices on the inner circle. (Variable numbers were assigned by the planning competition.) Figure 1(b) reveals at



(a) (b) (c) (d)

Figure 1. For the same driverlog CSP (a) an uninformative constraint graph plots variables on the circumference of a circle while (b) another constraint graph reveals more of its structure. (c) The adjacency matrix of the same graph (d) Subproblems, identified by local search, that significantly improve global search performance.

least six major, tightly-connected subproblems (darker edges) with looser (lighter) connections between subproblems.

Figure 1(c) is an alternative visualization of the same constraint graph as an adjacency matrix. With the lower-left corner as the origin, the x-axis and y-axis both record variable numbers. A point plotted at (x, y) represents a constraint between variables numbered x and y, where $x < y$. Darker points denote tighter constraints and lighter points looser ones. In addition to the structural information that Figure 1(b) provides, Figure 1(c) makes clear that the tight subproblems form a secondary structure: a path. In Figure 1(c), subproblems are loosely connected only to neighboring subproblems.

Based on such observed structural knowledge, we have designed a hybrid search algorithm. First it identifies the densely and tightly connected subproblems (*clusters*) shown circled in Figure 1(d), and then it solves the original problem guided by this structural knowledge (Epstein and Li, 2009b). This algorithm significantly outperforms a state-of-the-art search heuristic on real-world and benchmark CSPs. Indeed, in some cases search becomes an order of magnitude faster.

The next section of this paper provides background on traditional CSP search algorithms and heuristics. Subsequent sections explain how structural knowledge can guide search, describe the visualization program DrawCSP, and show the structure of various problems. Finally, we demonstrate improved search results with this structural knowledge.

Search for CSP Solutions

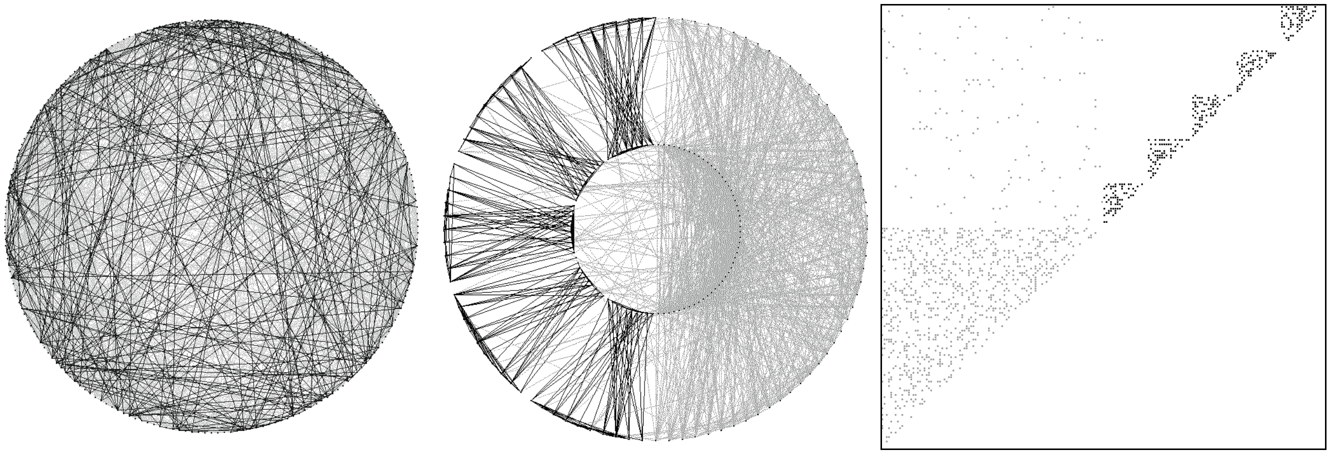
Global search for a solution to a CSP assigns a value to one variable at a time. Global search repeatedly tries to extend a *partial instantiation*, a set of value assignments, to a *full instantiation*, where every variable is assigned a value. A *solution* to a CSP is a full instantiation that satisfies all its constraints. During search, a *future* variable is one without an assigned value in the current partial instantiation. The (static) *degree* of a variable is the number of neighbors its vertex has in the constraint graph. The *dynamic degree* of a variable is the number of those neighbors that represent future variables.

After each new assignment, *propagation* removes from the domains of the future variables any values that are inconsistent with the current partial instantiation, thus producing *dynamic domains*. When a dynamic domain becomes empty (a *wipeout*) search *backtracks*, that is, it retracts previously assigned values and restores previously-reduced dynamic domains. All experiments reported here use MAC-3 propagation to maintain arc consistency (Sabin and Freuder, 1997) and chronological backtracking.

Global search is *complete* because it either finds a solution to a CSP or proves that no solution exists. A problem is labeled "solved" if search achieves either. CSP search is usually evaluated by time (in CPU seconds) and by the number of *nodes* (partial instantiations) explored.

Global search traditionally uses variable-ordering heuristics to select which variable to instantiate next. Many variable-ordering heuristics follow the *fail first* principle: address first those variables for which it is difficult to find values that lead to a solution (Haralick and Elliot, 1980). For example, a variable with a small domain and many future variables as neighbors is likely to be involved in a wipeout. *MinDomDeg*, the most popular traditional heuristic, accordingly prefers variables that minimize the ratio of their dynamic domain size to their dynamic degree. Because *MinDomDeg* overlooks constraints' tightness, however, it can perform poorly on some CSPs with special structure, as shown below in the experimental results.

Learning can improve the quality of search heuristics when it identifies troubled variables dynamically. One way to do so is to assign a weight, initialized to 1, to every con-



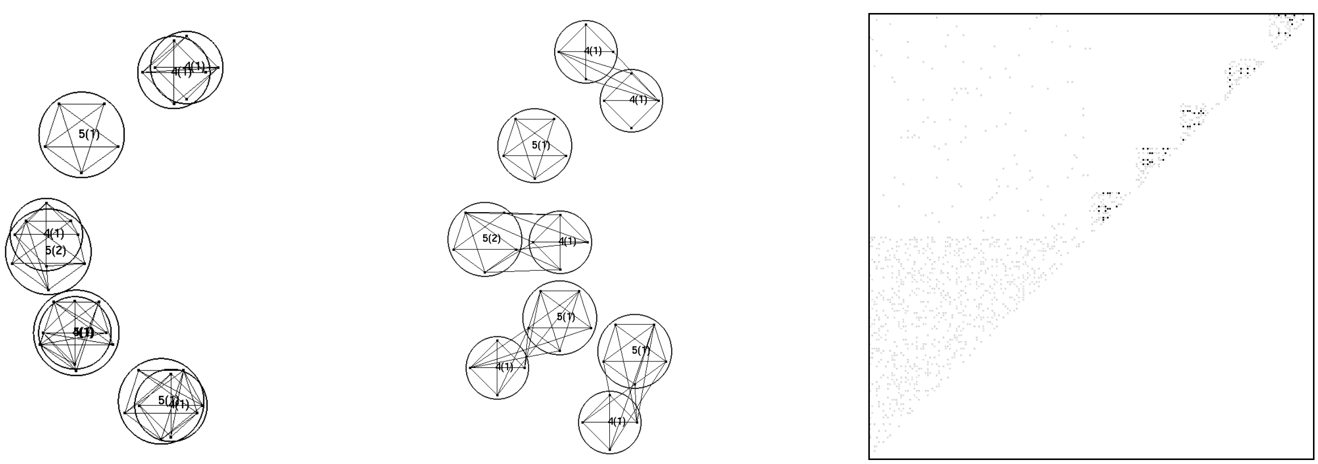
(a) (b) (c)

Figure 2. Constraint graphs for a CSP from the class *Comp*: (a) an uninformative constraint graph (b) the actual structure of the problem showing a large, loose central component on the right and five small but tight satellites on the left (c) the adjacency matrix constraint graph shows the central component at the lower left, the smaller satellites along the diagonal and, in the upper left, the loose links from the central component to its satellites, which are not connected to one another.

straint (Boussemart et al., 2004). Whenever, through propagation during search, a constraint causes a wipeout on a variable in its scope, that constraint's weight is increased by 1. The variable-ordering heuristic *MinDomWdeg* minimizes the ratio of dynamic domain size to *weighted degree* (the sum of the weights on all constraints whose scope includes the variable). As some constraints' weights gradually increase during search, *MinDomWdeg* favors variables in the scope of constraints that cause more wipeouts. At the beginning of search, however, *MinDomWdeg* is identical to *MinDomDeg,* because all weights are 1. Therefore early search guided by *MinDomWdeg* is just like that guided by *MinDomDeg*. Because it learns, *MinDomWdeg* can eventually recover from poor initial choices as search proceeds. Such recovery would be unnecessary, however, if these heuristics did not ignore tightness and structure.

Structure-guided search

A *structured* CSP has non-random characteristics that can be exploited by a structure-sensitive method to outperform a more general one. For example, a *composed* CSP is an artificial problem that partitions its variables into connected components: one *central component* connected by constraints (*links*) to at least one *satellite.* There are also constraints within satellites but no constraints between them. A class of composed problems has signature



(a) (b) (c)

Figure 3. *Foretell* finds clusters in the problem shown in Figure 2. Each cluster is circled; the label *x*(*y*) denotes that it includes *x* variables and that it needs *y* additional edges to become a clique. (a) Direct output from DrawCSP, where the five groups of clusters correspond to the five satellites (b) The nine clusters manually rearranged (c) Cluster variables are darkened in the adjacency matrix.

$<n,k,d,t>\ s\ <n',k',d',t'>\ d''\ t''$

This specifies a central component $<n,k,d,t>$ on n variables with maximum domain size k, density d, and tightness t, plus s satellites $<n',k',d',t'>$ each on n' variables with maximum domain size k', density d', and tightness t', and links between a satellite and the central component with density d'' and tightness t''. For example, Figure 2 shows an unsolvable problem in the composed class

Comp = <100,10,0.15,0.05>5<20,10,0.25,0.50>0.012, 0.05

Structure-guided search first uses a local search algorithm, *Foretell*, to identify clusters, and then guides global search for a solution with *Focus*, a cluster-oriented variable-ordering heuristic (Epstein and Li, 2009a). Figure 3 shows the clusters found by *Foretell* for the same *Comp* problem shown in Figure 2. The cluster finder *Foretell* was inspired by state-of-the-art work for both speed and accuracy on the DIMACS maximum clique problems (Hansen, Mladenovic and Urosevic, 2004). (A *clique* is a graph with density 1, that is, it has an edge between every pair of variables.) *Foretell* searches for large, dense and tight subproblems that either are cliques or near-cliques. (A *near-clique* is a graph with density very close to 1. Note, for example, the missing edges in the clusters of Figure 3(b).) Whereas the maximum clique algorithm relies on variable degree, *Foretell* relies on the notion of pressure. The *pressure* on a variable v, given all the constraints upon it, is the probability that when one of v's neighbors is assigned a value, at least one value will be excluded from v's domain. For a constraint with tightness t between variables V_1 and V_2 with domain sizes D_1 and D_2, respectively, *Foretell* calculates the initial pressure on variable V_1 as:

$$p(V_1) = \frac{1}{\text{degree}(V_1)} \sum_{c \in C \text{ on } V_1} \frac{\binom{(D_1-1)\cdot D_2}{(1-t)D_1 \cdot D_2}}{\binom{D_1 \cdot D_2}{(1-t)D_1 \cdot D_2}} \qquad [1]$$

Equation [1] estimates variable pressure to avoid the expense of precise calculation, with a correction to avoid bias in favor of variables with high degrees.

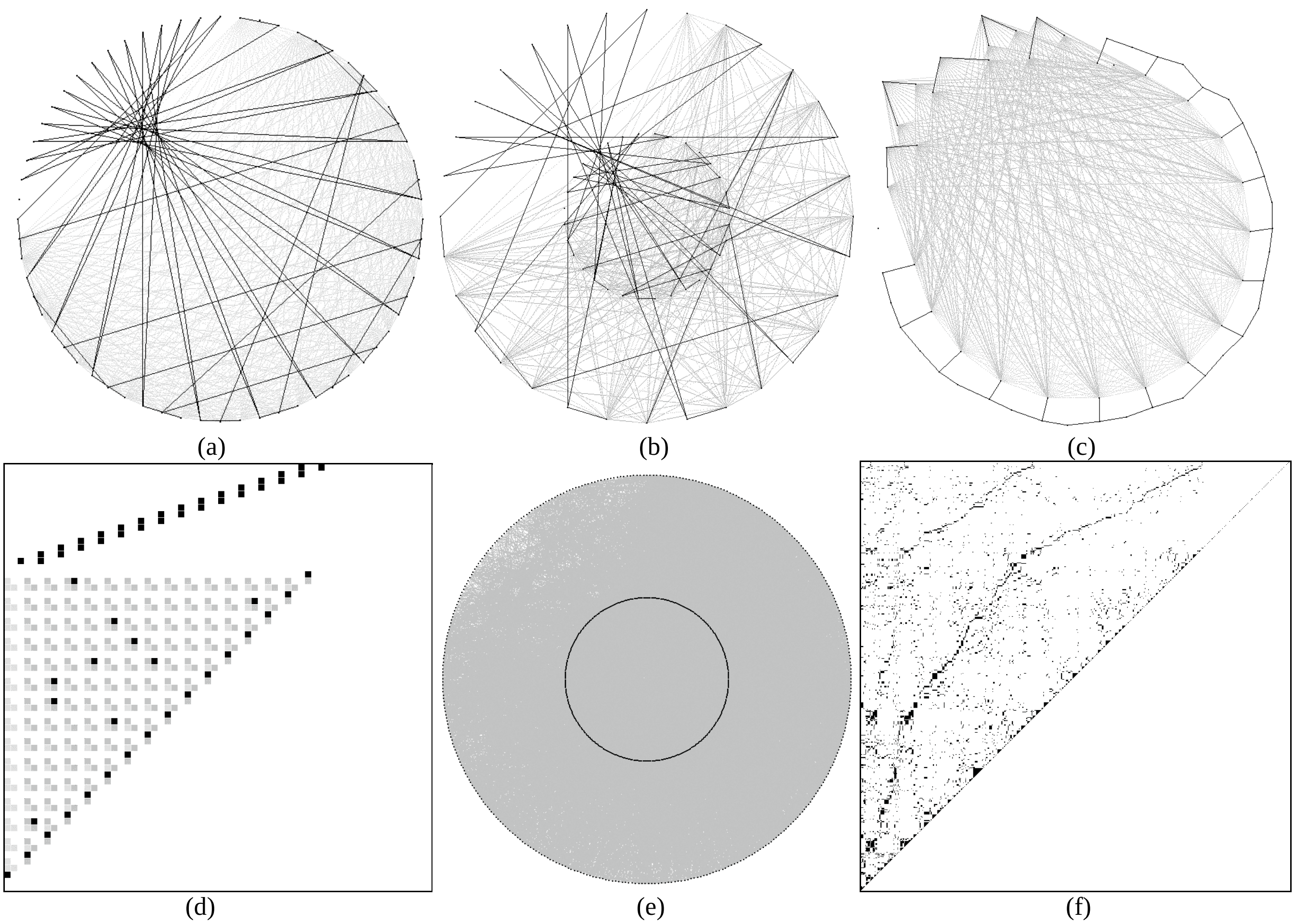


Figure 4. Examples of DrawCSP. Constraint graphs for BlackHole-4-4-e-0_ext.xml (a) with variables arranged on a single circle or (b) arranged on two circles and (c) after manual arrangement to reveal its structure, which includes a large tree and 5 short paths of length 2. (d) BlackHole-4-4-e-0_ext.xml displayed in the adjacency matrix mode. (e) Constraint graph on two concentric circles for a large optical network problem and (f) its adjacency matrix.

Once *Foretell* detects the clusters, *Focus* restricts search to one cluster at a time, and uses *MinDomWdeg* to break ties within that cluster. *Focus* selects a cluster with the minimum ratio of dynamic domain size to original domain size for all future variables in the cluster. This ratio dynamically estimates the extent to which tuples have been eliminated as possible cluster solutions, another application of the fail-first principle. Table 1 demonstrates that structure-guided search significantly outperforms both *MinDomDeg* and the adaptive *MinDomWdeg*.

Related structure-based work in CSP includes identification and exploitation of tractable (Dechter and Pearl, 1987; Gyssens, Jeavons and Cohen, 1994; Mackworth and Freuder, 1985) and complex structures (Gompert and Choueiry, 2005). Unlike clusters, however, that work ignores tightness along individual constraints, the crucial distinction between the satellites and the central component in a *Comp* problem.

DrawCSP, a CSP visualization program

DrawCSP is a small, portable CSP visualization program, written in C++ with OpenGL and the OpenGL Utility Toolkit (GLUT). DrawCSP accepts a CSP in XCSP format (Roussel and Lecoutre, 2008), which is essentially a variant of XML. DrawCSP uses the XML parser from (Berghen, 2009). There are two drawing modes; the graph mode that produced Figures 2(a) and 2(b), and the adjacency matrix mode that produced Figure 2(c).

In the graph mode, DrawCSP variables can be plotted either on one circle or on two concentric circles. The two-circle arrangement sometimes displays constraints between geometrically close variables more effectively than the single circle arrangement, as it did in Figures 1(b) and 2(b). The user can also manually move variables on the screen, and variables' coordinates can be saved to files for future re-display. Figures 4(a-c) show a black hole CSP (Gent et al., 2007) plotted on one circle, on two concentric circles, and after manually rearrangement.

In the adjacency matrix mode, a constraint with scope variables numbered x and y, $x < y$, is displayed with coordinates (x, y). Manual rearrangement of constraints is not allowed in matrix mode. Although a constraint graph represents a CSP, it can be difficult to detect the actual structure when the number of constraints is large or the density of the CSP is high. Because constraints do not overlap on the adjacency matrix presentation, an adjacency matrix may better clarify relationships and thereby provide more structural information. Figure 4(d), the adjacency matrix of the same problem shown in Figures 4(a-c), shows the problem's structure without any manual re-arrangement. Figures 4(e) and (f) are, respectively, the double-circle constraint graph and the adjacency matrix of a large optical network problem with 778 variables and 12876 constraints. No structure is visible in its constraint graph, but Figure 4(f) suggests that this problem has approximately three long paths of variables.

DrawCSP visualizes clusters when a cluster formation file (generated by the solver during search) is provided. If it is in graph mode, DrawCSP draws only the clusters, with the radius of each cluster proportional to the number of variables it contains. The center of each cluster's circle is the geometric center of all the vertices it includes, with their coordinates computed from what would have been their single-circle locations. The geometric center is selected to preserve the relative positions of the subproblems, as presented by clusters, in the entire CSP graph. For example, the clusters shown in Figure 1(d) closely represent their corresponding subproblems in Figure 1(b). Supported manual rearrangement of clusters includes translation and rotation. In adjacency matrix mode, DrawCSP plots constraints that are inside clusters darker than out-of-cluster constraints (Figure 3(c)).

Table 1: Structure-guided search speeds traditional heuristics on 50 *Comp* problems by more than an order of magnitude. Average and standard deviation are shown for search tree nodes and for time in CPU seconds, including time for cluster detection. Search is restricted to 1800 seconds. *MinDomDeg* fails to solve all but two instances; the other two heuristics solve all 50 problems. Boldface denotes results that are statistically significantly better at the 95% confidence level.

Heuristic	**Time**		**Nodes**	
MinDomDeg	1728.16	(355.88)	285751.97	(61368.70)
MinDomWdeg	83.58	(38.96)	12519.36	(5811.37)
Foretell + Focus	**4.31**	(2.41)	**497.96**	(324.33)

Visualization and search results

The experiments reported in this section compare the state-of-the-art learning heuristic *MinDomWdeg* against *Foretell* with *Focus*, and use *MinDomWdeg* to break ties within clusters. To control for the vagaries of local search, experiments with *Foretell* are always averaged across 10 trials for each problem. All experiments were run with the constraint solver *ACE* (Epstein, Freuder and Wallace, 2005). On each problem, *Foretell* was given a short time limit, usually less than 500 milliseconds, to find a cluster. *Foretell* identified as many clusters as it could until it could not find a cluster of size at least 3. The time that *Foretell* uses to find clusters is included in the performance measurements in Table 2.

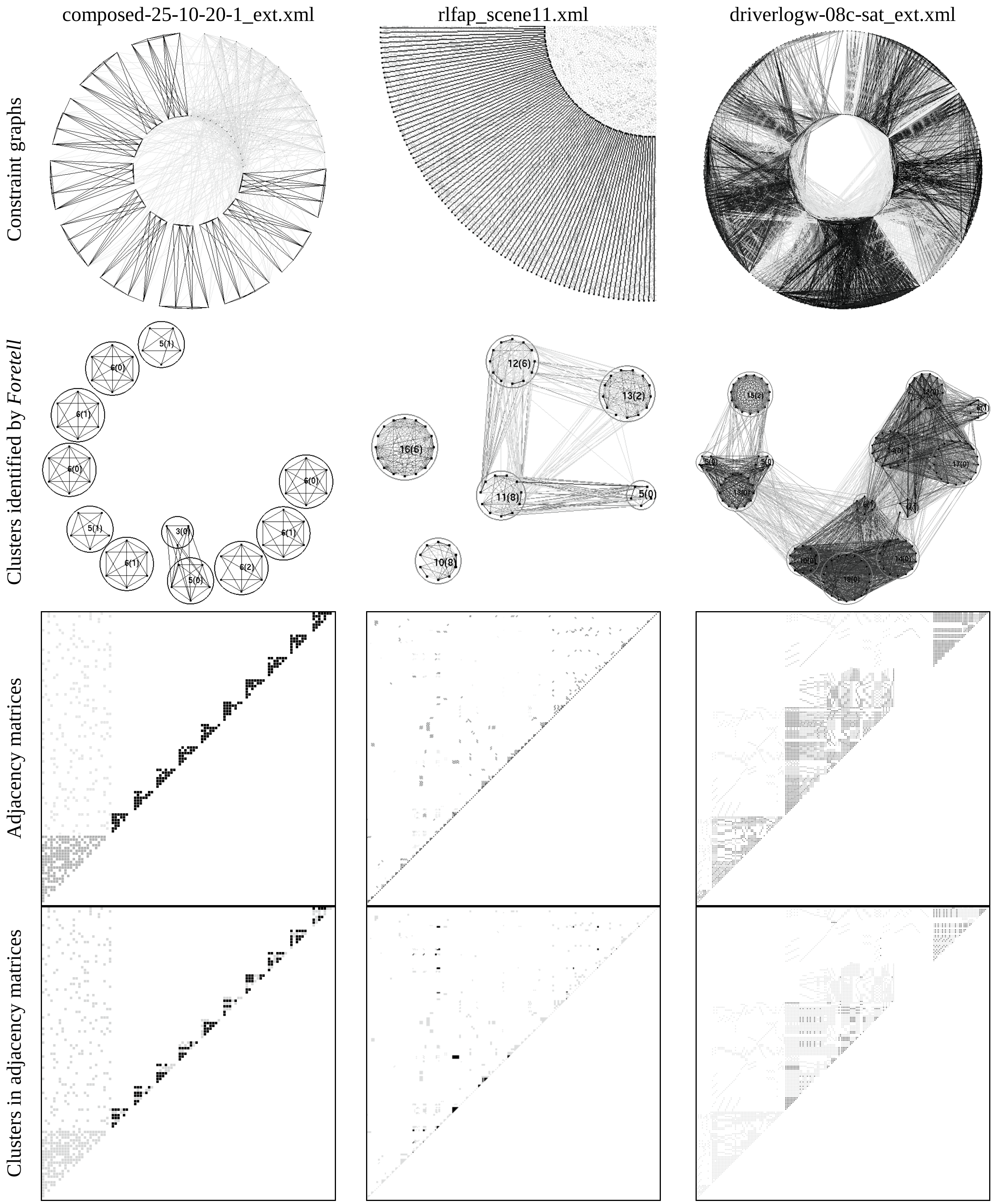


Figure 5. Constraint graphs, adjacency matrices and identified structures for a problem in composed-25-10-20-1_ext.xml, and for rlfap_scene11.xml and driverlogw-08c-sat_ext.xml. For clarity, only the lower-left quarters of the figures are shown for rlfap_scene11.xml's cluster graph, adjacency matrix and clusters within its adjacency matrix, and for driverlogw-08c-sat_ext.xml's adjacency matrix and clusters within its adjacency matrix.

Heuristics were compared on all the classes of composed problems on the benchmark website (Lecoutre, 2009), where there are 10 problems per class. A problem described there by *a-b-c* denotes a central component with *a* variables, *b* satellites of 8 variables each and *c* links. All variables have domain size 10, with constraint tightness 0.150 within the central component and 0.050 on each link. Constraint density within a satellite is always 0.786. The upper section of Table 2 shows the results on these classes (together with *Comp*). On seven classes, structure-guided search provided an order of magnitude speedup.

The lower section of Table 2 shows the results on the three most difficult driverlog problems and on scene11, a Radio Link Frequency Assignment Problem (*RLFAP*) (Murphey, Pardalos and Resende, 1999). An RLFAP involves hundreds of radio broadcasting facilities with limited broadcasting frequencies. If the frequencies of two geographically close broadcasting facilities are too close numerically, their signals will interfere with one another and communication will be distorted. Thousands of such pairs of broadcasting facilities in France are susceptible to interference. An RLFAP represents radio broadcasting facilities as variables, and restricts their frequencies with constraints between pairs of variables. The domain of an RLFAP variable is the set of radio frequencies that a facility can be assigned. A solution to an RLFAP CSP is equivalent to the establishment of interference-free communication. Scene11 is the most difficult solvable RLFAP problem.

While these pictures support improved search performance, they also leave many questions for future investigation. For example, the RLFAP problem has tight constraints between every pair V_i and V_{i+1} where i is an even number. This is why the double-circle constraint graph resembles a sun with rays of light. The clusters that *Foretell* detects are also bipartite on tight edges. It would be interesting to study whether the structure of evenly distributed tight constraints makes this problem difficult. As another example, the constraint graph and the cluster graph of driverlogw-08c in Figure 5 show that a subproblem can be covered by more than one cluster. This observation raises questions about the possibility that some order should be imposed on how subsets of clusters are visited during search.

The semantics of tight subproblems (darker) in a constraint graph are only sometimes available for inspection. For example, composed problems were designed and generated to mislead traditional variable-ordering heuristics, such as *MinDomDeg*, to the loose central component, although the actual contention is in the tight satellites. Semantics for the tight subproblems in the driverlog problems are not provided. *Foretell*, however, does not rely on problem semantics. It looks instead for structure that is likely to be contentious during search. When the problem is solvable, *clusters* provide guidance for a more effective search. When the problem is unsolvable, *clusters* provide an accessible description of the unsolvable core and thereby suggest areas that might be *relaxed*, that is, made easier to satisfy, so that the problem has a solution. Relaxation could add values to the domains of core variables or accept more value pairs for core constraints.

Conclusion

This paper presents DrawCSP, a visualization program for constraint satisfaction problems. DrawCSP can display a

Table 2: Preference for variables in clusters improves search. Problem identifiers above the line are from (Lecoutre, 2009). For example, the first class is <25, 10, 0.67, 0.15> 10 <8, 10, 0.79, 0.50> 0.01, 0.05. Full descriptions are available in (Lecoutre, Boussemart and Hemery, 2004). All data for *Foretell* with *Focus* are averaged across 10 runs, and show mean and standard deviation. At the 95% confidence level, *Foretell* + *Focus* outperforms *MinDomWdeg* on these problems. Order of magnitude improvements over *MinDomWdeg* are in boldface.

Problems	***d***	***t′***	***d″***	***Number of instances***	***MinDomWdeg***		***Foretell* + *Focus* Time**		***Foretell* + *Focus* Nodes**	
					Time	**Nodes**	μ	σ	μ	σ
25-10-20	0.667	0.50	0.010	10	2.49	670.10	0.88	0.47	192.07	149.88
25-1-80	0.667	0.65	0.010	10	0.95	308.00	0.26	0.25	94.50	71.81
75-1-80	0.216	0.65	0.133	10	2.32	595.20	0.37	0.17	181.40	21.69
25-1-2	0.667	0.65	0.010	10	1.01	553.00	**0.02**	0.00	**41.40**	1.36
25-1-25	0.667	0.65	0.125	10	0.91	465.70	**0.04**	0.02	**41.60**	1.29
25-1-40	0.667	0.65	0.200	10	1.10	473.80	**0.07**	0.02	**41.50**	1.21
75-1-2	0.216	0.65	0.003	10	3.33	1171.70	**0.04**	0.01	**91.60**	1.50
75-1-25	0.216	0.65	0.042	10	3.29	1084.40	**0.15**	0.12	**91.40**	1.29
75-1-40	0.216	0.65	0.067	10	2.97	960.90	**0.15**	0.14	**91.30**	1.28
Comp	0.150	0.50	0.120	50	83.58	12519.40	**4.31**	2.41	**497.96**	324.33
RLFAP scene 11	—	—	—	1	40.74	2810.00	16.00	0.07	978.00	0.00
Driverlogw 02	—	—	—	1	19.49	1862.00	6.08	2.14	507.80	223.11
Driverlogw 08c	—	—	—	1	101.58	4820.00	21.09	0.28	967.80	0.42
Driverlogw 09	—	—	—	1	634.03	15987.00	303.52	4.31	7847.10	153.74

CSP in two different ways. Each of them provides a direct view of the problem that may help design structure-oriented search algorithms for its solution. DrawCSP can also display the structure identified by solvers like ACE in a picture that users can easily verify and compare with the problem's original structure.

Visualization of identified structure supports the design and improvement of search heuristics that exploit such structure. The constraint graph, the adjacency matrix, and the cluster graphs provide abstractions of the problem. They improve users' understanding of the internal structure and suggest possible alternative ways to model the problem if it proves too costly to solve. The structure observed in DrawCSP output inspired the design of a structure-guided search algorithm for constraints solvers, one which achieves significant performance improvement over a state-of-the-art search heuristic on structured problems.

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