Computational Vision

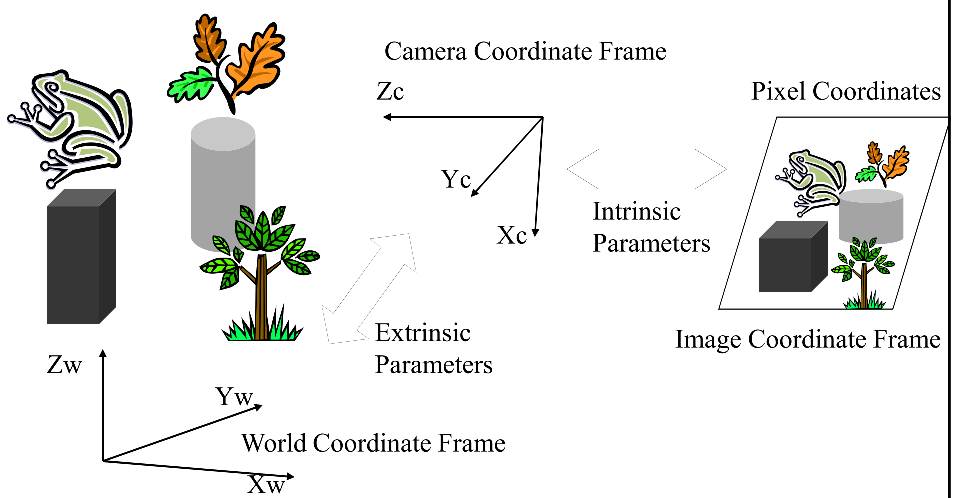
Camera Calibration

Trucco, chapter 6

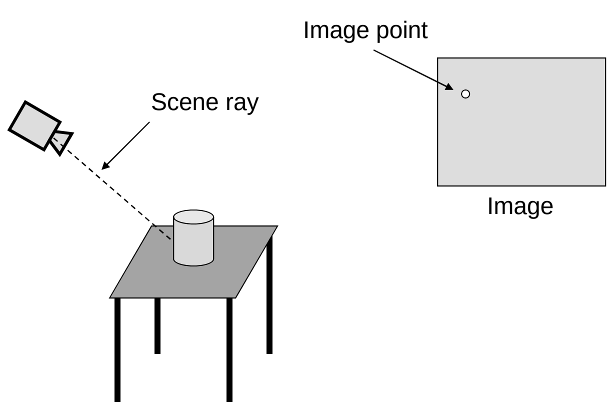
Camera Calibration

- Problem: Estimate camera's extrinsic & intrinsic parameters.
- Method: Use image(s) of known scene.
- Tools:
  - Geometric camera models.
  - SVD and constrained least-squares.
  - Line extraction methods.

Coordinate Frames

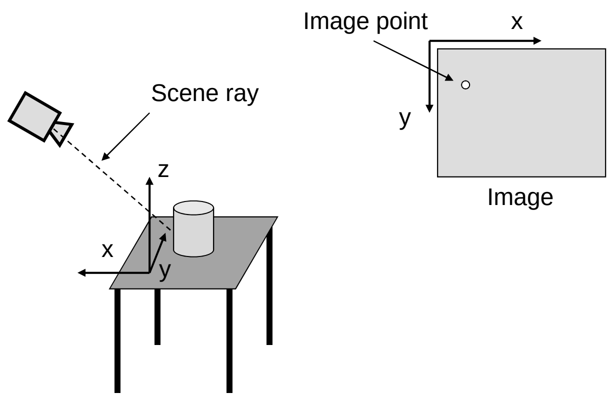


Why Calibrate?



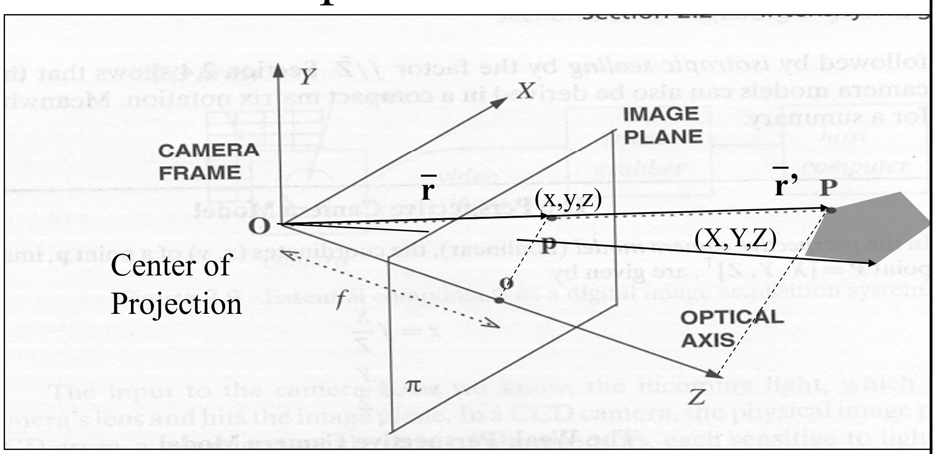
Calibration: relates points in the image to rays in the scene

Why Calibrate?



Calibration: relates points in the image to rays in the scene

Perspective Camera



$\bar{r} = (x,y,z)$

$\bar{r}' = (X,Y,Z)$

$\bar{r}/f = \bar{r}'/Z$

f: effective focal length: distance of image plane from O.

x=f * X/Z

y=f * Y/Z

z=f

Extrinsic Parameters

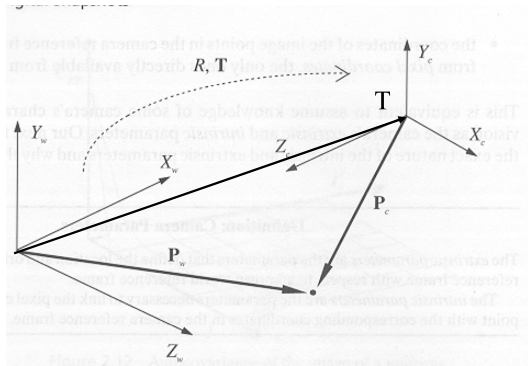


Figure 2.13 The relation between camera and world coordinate frames.

$$P_c = R(P_w - T)$$

Translation followed by rotation

Extrinsic Parameters (2nd formulation)

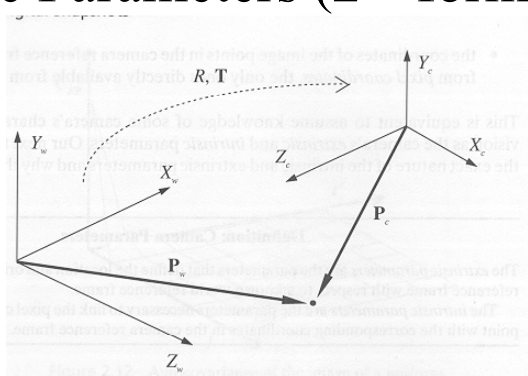


Figure 2.13 The relation between camera and world coordinate frames.

$$P_c = R\,P_w + T$$

R same as before
T different

Rotation followed by translation

The Rotation Matrix

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}.$$

$R * R^T = R^T * R = I$ =>
$R^{-1} = R^T$
Orthonormal Matrix
Degrees of freedom?

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Intrinsic Parameters

$$x = \frac{f}{Z} X$$

$$y = \frac{f}{Z} Y$$

The Transformation between Camera and Image Frame Coordinates

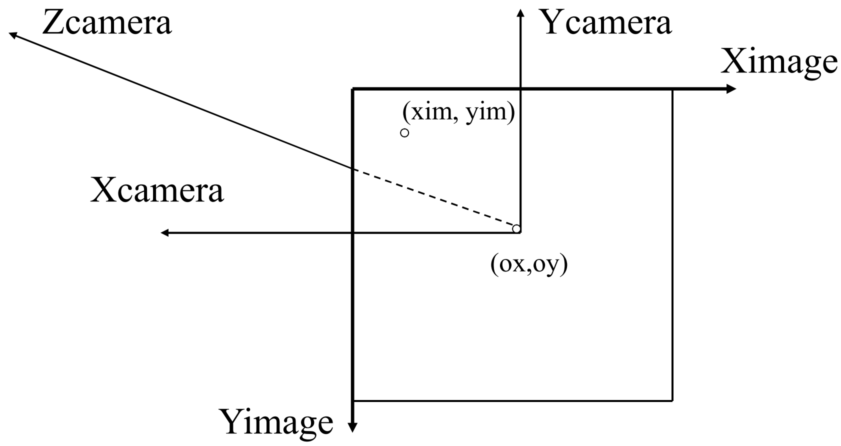
Neglecting any geometric distorsions possibly introduced by the optics and in the assumption that the CCD array is made of a rectangular grid of photosensitive elements, we have

$$x = -(x_{im} - o_x)s_x$$
$$y = -(y_{im} - o_y)s_y \quad (2.20)$$

with (o_x, o_y) the coordinates in pixel of the image center (the principal point), and (s_x, s_y) the effective size of the pixel (in millimeters) in the horizontal and vertical direction respectively.

Therefore, the current set of intrinsic parameters is f, o_x, o_y, s_x, s_y.

Image and Camera Frames



Geometric Model

x= $-(x_{im} - o_x)s_x = f \frac{X_c}{Z_c}$

y= $-(y_{im} - o_y)s_y = f \frac{Y_c}{Z_c}$

3D Point in Camera Coordinate Frame

- Transformation from Image to Camera Frame. (ox,oy,sx,sy)
- No distortion!

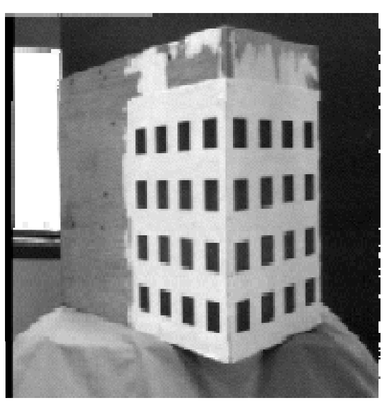
- Transformation from World to Camera Frame.
- Perspective projection (f, R, T)

Point in Camera Frame

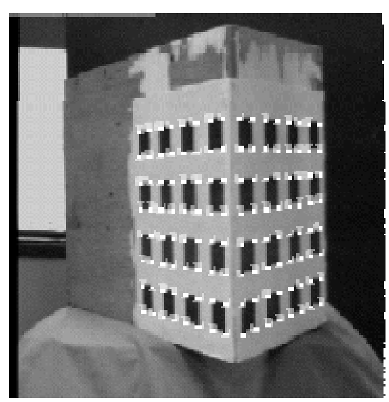
Camera Calibration: Issues

- Which parameters need to be estimated.
  - Focal length, image center, aspect ratio
  - Radial distortions
- What kind of accuracy is needed.
  - Application dependent
- What kind of calibration object is used.
  - One plane, many planes
  - Complicated three dimensional object

Camera Calibration

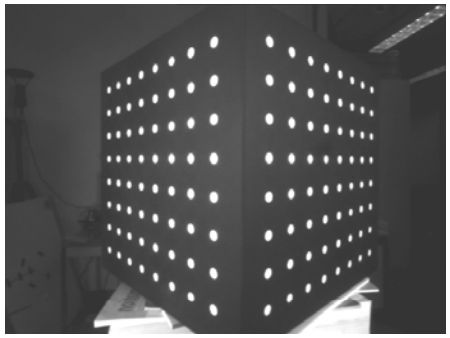


Calibration object



Extracted features

Camera Calibration



Extract centers of circles

Basic Equations

$$x_{im} = -\frac{f}{s_x}\frac{X^c}{Z^c} + o_x$$

$$y_{im} = -\frac{f}{s_y}\frac{Y^c}{Z^c} + o_y$$

$$X^c = r_{11}X^w + r_{12}Y^w + r_{13}Z^w + T_x$$

$$Y^c = r_{21}X^w + r_{22}Y^w + r_{23}Z^w + T_y$$

$$Z^c = r_{31}X^w + r_{32}Y^w + r_{33}Z^w + T_z$$

Basic Equations

$$x = -f_x \frac{X^c}{Z^c} + o_x$$

$$y = -f_y \frac{Y^c}{Z^c} + o_y$$

$$X^c = r_{11} X^w + r_{12} Y^w + r_{13} Z^w + T_x$$

$$Y^c = r_{21} X^w + r_{22} Y^w + r_{23} Z^w + T_y$$

$$Z^c = r_{31} X^w + r_{32} Y^w + r_{33} Z^w + T_z$$

Basic Equations

$$x = -f_x \frac{X^c}{Z^c} + o_x$$

$$y = -\frac{f_x}{\alpha} \frac{Y^c}{Z^c} + o_y$$

$$X^c = r_{11} X^w + r_{12} Y^w + r_{13} Z^w + T_x$$

$$Y^c = r_{21} X^w + r_{22} Y^w + r_{23} Z^w + T_y$$

$$Z^c = r_{31} X^w + r_{32} Y^w + r_{33} Z^w + T_z$$

Basic Equations

Extrinsic Parameters

1) Rotation matrix R (3x3)
2) Translation vector **T** (3x1)

Intrinsic Parameters

1) fx=f/sx, length in effective horizontal pixel size units.
2) α=sy/sx, aspect ratio.
3) (ox,oy), image center coordinates.
4) Radial distortion coefficients.

Total number of parameters (excluding distortion): 8

Basic Equations

$$x - o_x = -f_x \frac{r_{11} X^w + r_{12} Y^w + r_{13} Z^w + T_x}{r_{31} X^w + r_{32} Y^w + r_{33} Z^w + T_z}$$

$$y - o_y = -f_y \frac{r_{21} X^w + r_{22} Y^w + r_{23} Z^w + T_y}{r_{31} X^w + r_{32} Y^w + r_{33} Z^w + T_z}$$

1) **Assume that image center is known.**
2) **Solve for the remaining parameters.**
3) **Use N image points** (x_i, y_i) **and their corresponding N world points** $[X_i^w, Y_i^w, Z_i^w]^T$

Basic Equations

$$x = -f_x \frac{r_{11} X^w + r_{12} Y^w + r_{13} Z^w + T_x}{r_{31} X^w + r_{32} Y^w + r_{33} Z^w + T_z}$$

$$y = -f_y \frac{r_{21} X^w + r_{22} Y^w + r_{23} Z^w + T_y}{r_{31} X^w + r_{32} Y^w + r_{33} Z^w + T_z}$$

(1)

1) **Assume that image center is known.**
2) **Solve for the remaining parameters.**
3) **Use N image points** (x_i, y_i) **and their corresponding**
   N world points $[X_i^w, Y_i^w, Z_i^w]^T$

Basic Equations

$$x_i f_y (r_{21} X_i^w + r_{22} Y_i^w + r_{23} Z_i^w + T_y)$$
$$=$$
$$y_i f_x (r_{11} X_i^w + r_{12} Y_i^w + r_{13} Z_i^w + T_x)$$

(2)

1) **Assume that image center is known.**
2) **Solve for the remaining parameters.**
3) **Use N image points** (x_i, y_i) **and their corresponding**
   N world points $[X_i^w, Y_i^w, Z_i^w]^T$

Basic Equations

$$x_i X_i^w v_1 + x_i Y_i^w v_2 + x_i Z_i^w v_3 + x_i v_4 - y_i X_i^w v_5 - y_i Y_i^w v_6 - y_i Z_i^w v_7 - y_i v_8 = 0 \quad \textbf{(3)}$$

$$v_1 = r_{21}, v_5 = \alpha r_{11}$$
$$v_2 = r_{22}, v_6 = \alpha r_{12}$$
$$v_3 = r_{23}, v_7 = \alpha r_{13}$$
$$v_4 = T_y, v_8 = \alpha T_x$$

Basic Equations

$$x_i X_i^w v_1 + x_i Y_i^w v_2 + x_i Z_i^w v_3 + x_i v_4 - y_i X_i^w v_5 - y_i Y_i^w v_6 - y_i Z_i^w v_7 - y_i v_8 = 0 \quad \textbf{(3)}$$

$$v_1 = r_{21}, v_5 = \alpha r_{11}$$
$$v_2 = r_{22}, v_6 = \alpha r_{12}$$
$$v_3 = r_{23}, v_7 = \alpha r_{13}$$
$$v_4 = T_y, v_8 = \alpha T_x$$

$$A\mathbf{v} = 0$$

How would we solve this system?

Basic Equations

$$A = \begin{bmatrix} x_1 X_1^w & x_1 Y_1^w & x_1 Z_1^w & x_1 & -y_1 X_1^w & -y_1 Y_1^w & -y_1 Z_1^w & -y_1 \\ x_2 X_2^w & x_2 Y_2^w & x_2 Z_2^w & x_2 & -y_2 X_2^w & -y_2 Y_2^w & -y_2 Z_2^w & -y_2 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ x_N X_N^w & x_N Y_N^w & x_N Z_N^w & x_N & -y_N X_N^w & -y_N Y_N^w & -y_N Z_N^w & -y_N \end{bmatrix}$$

$$A\mathbf{v} = 0 \qquad \textbf{(3)}$$

How would we solve this system?

Rank of matrix A?

Solution up to a scale factor.

Singular Value Decomposition

Appendix A.6

$$A = UDV^T$$

A: m x n
U: m x m, columns orthogonal unit vectors.
V: n x n , -//-
D: m x n , diagonal. The diagonal elements σi are the singular values σ1>= σ2>= … >= σn >= 0

Singular Value Decomposition

Appendix A.6

$$A = UDV^T$$

1. **Square A non-singular iff σi != 0**
2. **For square A C=σ1/σN is the condition number**
3. **For rectangular A # of non-zero σi is the rank**
4. **For square non-singular A:** $A^{-1} = VD^{-1}U^T$
5. **For square A, pseudoinverse:** $A^{+} = VD_0^{-1}U^T$
6. **Singular values of A = square roots of eigenvalues of** AA^T **and** A^TA
7. **Columns of U, V Eigenvectors of** AA^T A^TA
8. **Frobenius norm of a matrix**

Singular Value Decomposition

Appendix A.6

$$A = UDV^T$$

$$A\mathbf{v} = 0$$

If rank(A)=n-1 (7 in our case) then

the solution is the eigenvector which corresponds to the ONLY zero eigenvalue.

Solution up to a scale factor.

Solving for v

$$A = \begin{bmatrix} x_1 X_1^w & x_1 Y_1^w & x_1 Z_1^w & x_1 & -y_1 X_1^w & -y_1 Y_1^w & -y_1 Z_1^w & -y_1 \\ x_2 X_2^w & x_2 Y_2^w & x_2 Z_2^w & x_2 & -y_2 X_2^w & -y_2 Y_2^w & -y_2 Z_2^w & -y_2 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ x_N X_N^w & x_N Y_N^w & x_N Z_N^w & x_N & -y_N X_N^w & -y_N Y_N^w & -y_N Z_N^w & -y_N \end{bmatrix}$$

$$A\mathbf{v} = 0 \qquad \textbf{(3)}$$

How would we solve this system: SVD.

Solution: $\overline{\mathbf{v}} = \gamma(r_{21}, r_{22}, r_{23}, T_y, \alpha r_{11}, \alpha r_{12}, \alpha r_{13}, \alpha T_x)$

Uknown scale factor γ=?

Aspect ratio α=?

Solving for Tz and fx?

Solving for Tz and fx?

$$x_i(r_{31}X_i^w + r_{32}Y_i^w + r_{33}Z_i^w + T_z) = -f_x(r_{11}X_i^w + r_{12}Y_i^w + r_{13}Z_i^w + T_x)$$

$$A\begin{pmatrix} T_z \\ f_x \end{pmatrix} = \mathbf{b}$$

How would we solve this system?

Solving for Tz and fx?

$$x_i(r_{31}X_i^w + r_{32}Y_i^w + r_{33}Z_i^w + T_z) = -f_x(r_{11}X_i^w + r_{12}Y_i^w + r_{13}Z_i^w + T_x)$$

$$A\begin{pmatrix} T_z \\ f_x \end{pmatrix} = \mathbf{b}$$

How would we solve this system?

$$\begin{pmatrix} \hat{T}_z \\ \hat{f}_x \end{pmatrix} = (A^T A)^{-1} A^T \mathbf{b}$$

Solution in the least squares sense.

Camera Center

Camera Models (linear versions)

$$M_{int} = \begin{pmatrix} -f/s_x & 0 & o_x \\ 0 & -f/s_y & o_y \\ 0 & 0 & 1 \end{pmatrix}$$

Elegant decomposition.
No distortion!

$$M_{ext} = \begin{pmatrix} r_{11} & r_{12} & r_{13} & -\mathbf{R}_1^\top \mathbf{T} \\ r_{21} & r_{22} & r_{23} & -\mathbf{R}_2^\top \mathbf{T} \\ r_{31} & r_{32} & r_{33} & -\mathbf{R}_3^\top \mathbf{T} \end{pmatrix},$$

The Linear Matrix Equation of Perspective Projections

Homogeneous Coordinates

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = M_{int} M_{ext} \begin{pmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{pmatrix}.$$

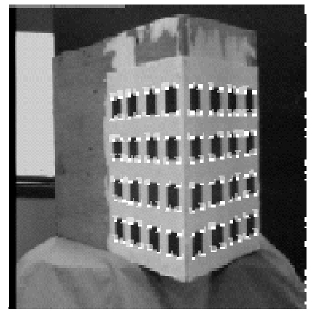
Measured Pixel (xim, yim) ⟵ World Point (Xw, Yw,Zw)

? ⟶

Camera Calibration – Other method

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = P \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$x = \frac{u}{w} \qquad y = \frac{v}{w}$$



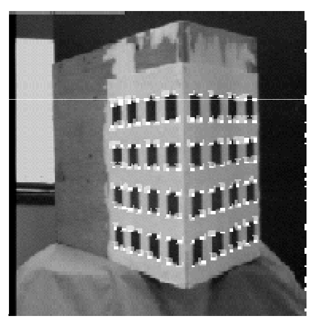
Extracted features

Step 1: Estimate P
Step 2: Decompose P into internal and external parameters R,T,C

Camera Calibration: Step 1

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = P \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$x = \frac{u}{w} \qquad y = \frac{v}{w}$$



Extracted features

$$wx = u$$
$$wy = v$$

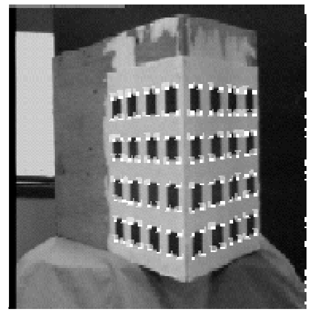
w

$$x(p_{31}X + p_{32}Y + p_{33}Z + p_{34}) = p_{11}X + p_{12}Y + p_{13}Z + p_{14} \leftarrow u$$
$$y(p_{31}X + p_{32}Y + p_{33}Z + p_{34}) = p_{21}X + p_{22}Y + p_{23}Z + p_{24} \leftarrow v$$

Each point (x,y) gives us two equations

Camera Calibration: Step 1

$$\begin{bmatrix} X & Y & Z & 1 & 0 & 0 & 0 & 0 & -xX & -xY & -xZ & -x \\ 0 & 0 & 0 & 0 & X & Y & Z & 1 & -yX & -yY & -yZ & -y \end{bmatrix} \begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$



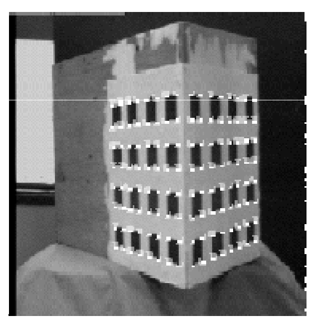
Extracted features

$$x(p_{31}X + p_{32}Y + p_{33}Z + p_{34}) = p_{11}X + p_{12}Y + p_{13}Z + p_{14}$$
$$y(p_{31}X + p_{32}Y + p_{33}Z + p_{34}) = p_{21}X + p_{22}Y + p_{23}Z + p_{24}$$

Each corner (x,y) gives us two equations

Camera Calibration: Step 1

$$2n \left\{ \underbrace{\begin{bmatrix} X & Y & Z & 1 & 0 & 0 & 0 & 0 & -xX & -xY & -xZ & -x \\ 0 & 0 & 0 & 0 & X & Y & Z & 1 & -yX & -yY & -yZ & -y \\ \cdots \\ \vdots \\ \cdots \\ X & Y & Z & 1 & 0 & 0 & 0 & 0 & -xX & -xY & -xZ & -x \\ 0 & 0 & 0 & 0 & X & Y & Z & 1 & -yX & -yY & -yZ & -y \end{bmatrix}}_{A} \right. \begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$



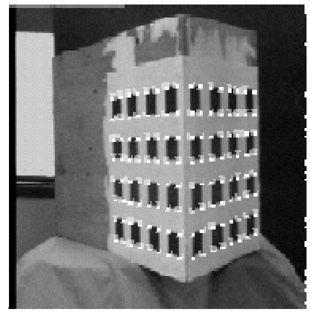
Extracted features

n points gives us 2n equations

Camera Calibration: Step 1

$$
2n\left\{\begin{bmatrix} X & Y & Z & 1 & 0 & 0 & 0 & 0 & -xX & -xY & -xZ & -x \\ 0 & 0 & 0 & 0 & X & Y & Z & 1 & -yX & -yY & -yZ & -y \\ \cdots \\ \vdots \\ \cdots \\ X & Y & Z & 1 & 0 & 0 & 0 & 0 & -xX & -xY & -xZ & -x \\ 0 & 0 & 0 & 0 & X & Y & Z & 1 & -yX & -yY & -yZ & -y \end{bmatrix}\right. \begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \\ p_{31} \\ p_{32} \\ p_{33} \\ p_{34} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}
$$

A



Extracted features

We need to solve $A\mathbf{p} = 0$

In the presence of noise we need to solve $\min_{\mathbf{p}} \|A\mathbf{p}\|$

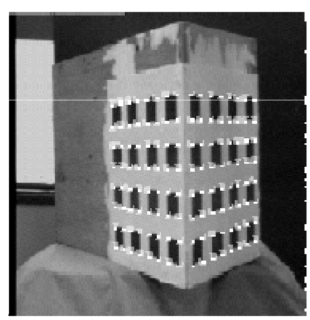
The solution is given by the eigenvector with the smallest eigenvalue of $A^T A$

Camera Calibration: Step 1

The result can be improved through non-linear minimization.

$$
\min_{\mathbf{p}} \sum_i \left(\left(x_i - \frac{u_i}{w_i} \right)^2 + \left(y_i - \frac{v_i}{w_i} \right)^2 \right)
$$

$$
\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = P \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}
$$



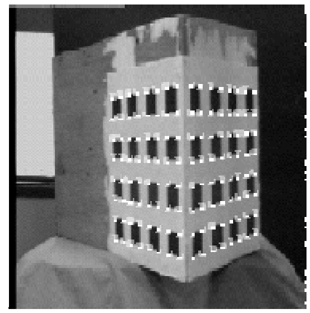
Extracted features

Camera Calibration: Step 1

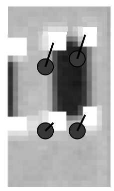
The result can be improved through non-linear minimization.

$$\min_{\mathbf{p}} \sum_i \left(\left(x_i - \frac{u_i}{w_i} \right)^2 + \left(y_i - \frac{v_i}{w_i} \right)^2 \right)$$

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = P \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$



Extracted features



Minimize the distance between the predicted and detected features.