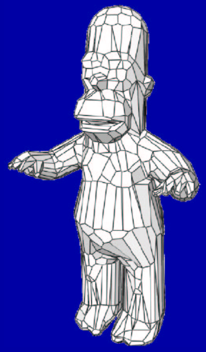


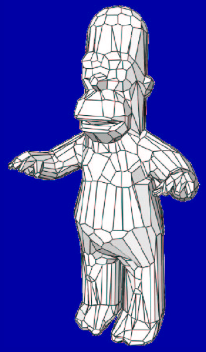
Variational Shape Approximation

David Cohen-Steiner

Pierre Alliez

Mathieu Desbrun

SIGGRAPH - 2004

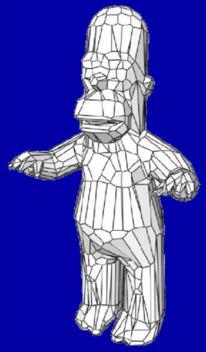


Introduction

Introduction

The Problem:

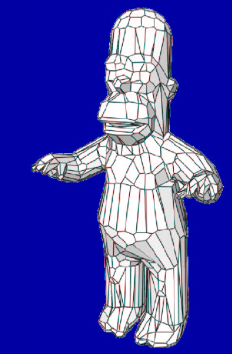
Introduction



Other approaches:

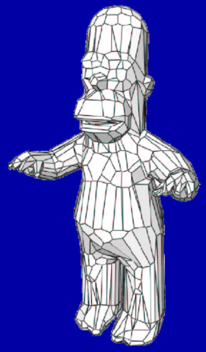
- Partitioning
  - Mesh decimation: Garland, ...
  - Greedy $\Longrightarrow$ Suboptimal meshes

Introduction



Other approaches:

- Partitioning
  - Mesh decimation: Garland, ...
  - Greedy $\Longrightarrow$ Suboptimal meshes
- Global Optimization
  - Energy Functional: Hoppe *et al.*
  - Irregular mesh, geometric efficiency



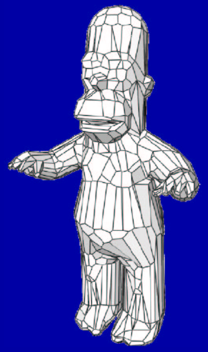
Introduction

Other approaches:

- Partitioning
  - Mesh decimation: Garland, ...
  - Greedy $\Longrightarrow$ Suboptimal meshes
- Global Optimization
  - Energy Functional: Hoppe *et al.*
  - Irregular mesh, geometric efficiency
- Anisotropy
  - Remeshing: Turk, Lee, Kobelt, ...
  - Quality of the mesh?

Shape Approximation

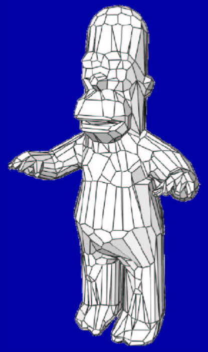
Shape Approximation



Approximation Theory:

- Functional Setting
  - Strong Results in $\mathcal{L}_p$: Splines, ...
  - Relies on Parameterization :-(

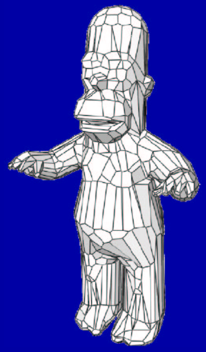
Shape Approximation



Approximation Theory:

- Height Field Approximation
  - Obvious Parameterization.
  - Few results about optimality in $\mathcal{L}_p$.
  - Few results on approximation of error.
  - Results are fairly narrow in scope
    - restricted to height fields,
    - triangulations are assumed to be interpolating the height field at vertices,
    - asymptotical case does not help for a small number of triangles.

Shape Approximation



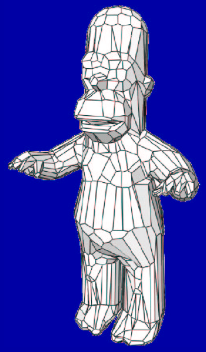
Approximation Theory:

- Arbitrary Geometry
  - Lack of knowledge on optimally efficient piecewise linear approximation.
  - Greedy Algorithms: (- triangles, ? error) or (- error, ? triangles)
  - Metric commonly used: $\boldsymbol{\mathcal{L}_p}$

$$\mathbf{d}_p(X,Y) = \left(\frac{1}{|X|} \int_{x \in X} \mathbf{d}_p(x,Y)^p \mathrm{d}x \right)^{\frac{1}{p}}$$

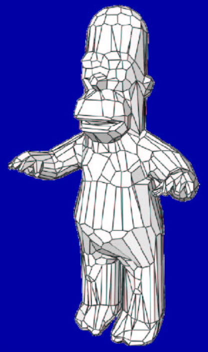
$$\mathbf{d}_p(x,Y) = \inf_{y \in Y} \|x - y\|_p$$

Shape Approximation



Variational Partitioning and Proxies

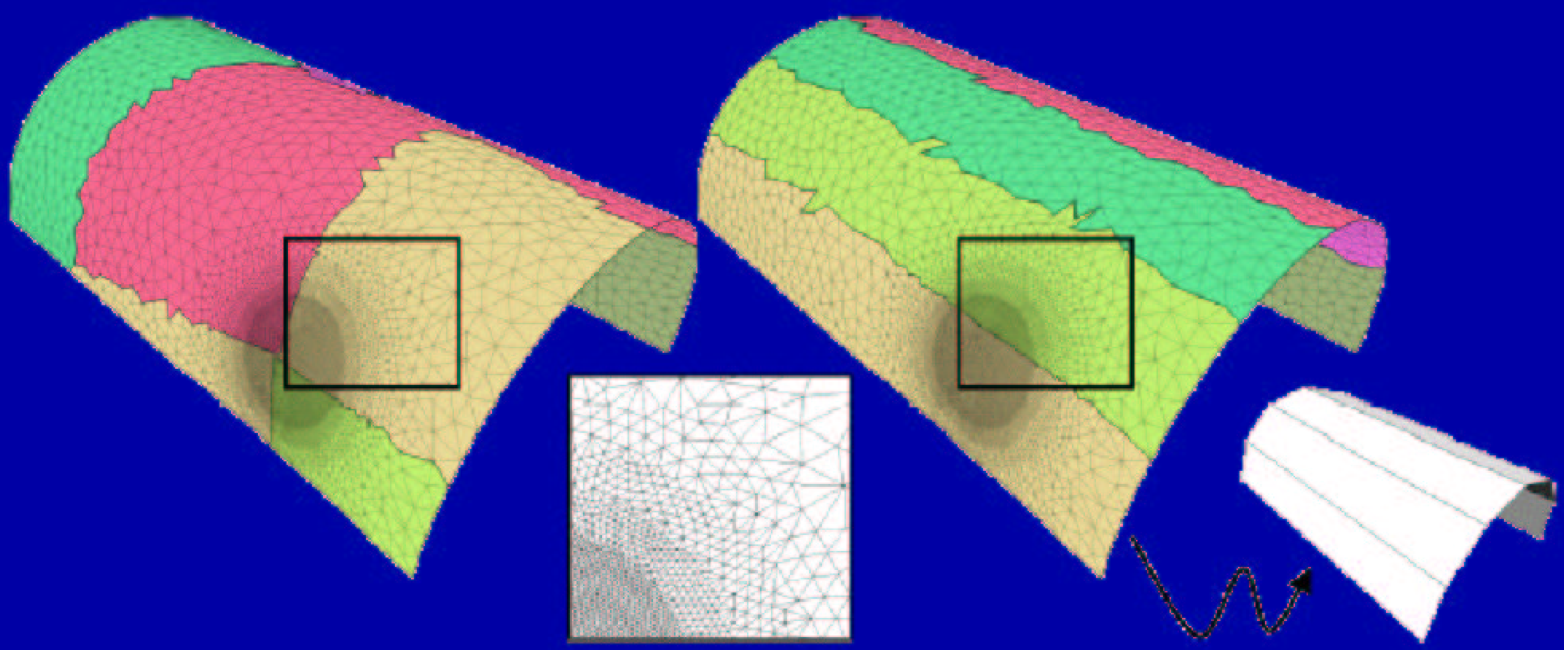
- Remove topology from the search
- Partitioning
  - iteratively seek a partition that minimizes a given error metric.
- Partition and proxies
  - Given a mesh partition $R = \{R_i\}$
  - A shape proxy is a pair $P_i = (\mathbf{x}_i, \mathbf{n}_i)$, where
    $\mathbf{x}_i \longleftarrow$ "average" Point
    $\mathbf{n}_i \longleftarrow$ "average" Normal

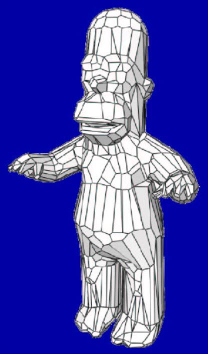


Shape Approximation

Metrics on Proxies

$$\mathbf{d}_2(R_i, P_i) = \int_{x \in R_i} \|x - \Pi_i(x)\|^2 \mathrm{d}x$$

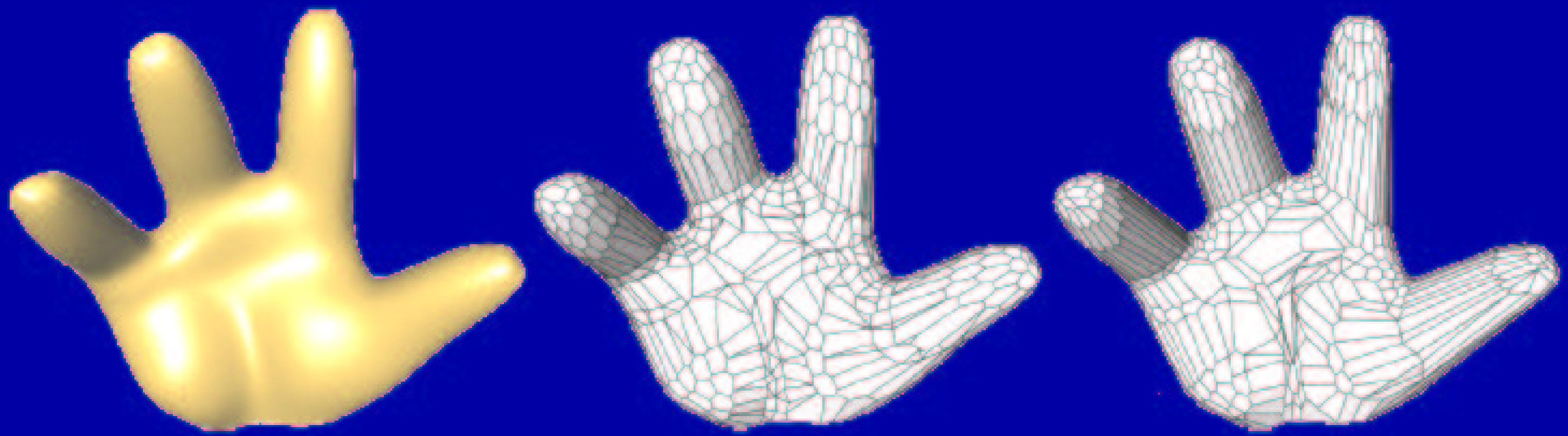


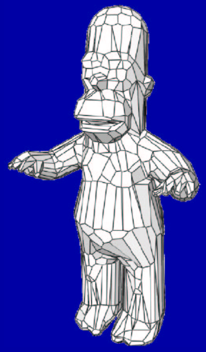


Shape Approximation

Metrics on Proxies

$$\mathbf{d}_{2,1}(R_i, P_i) = \int_{x \in R_i} \|\mathbf{n}(x) - \mathbf{n}_i\|^2 \mathrm{d}x$$

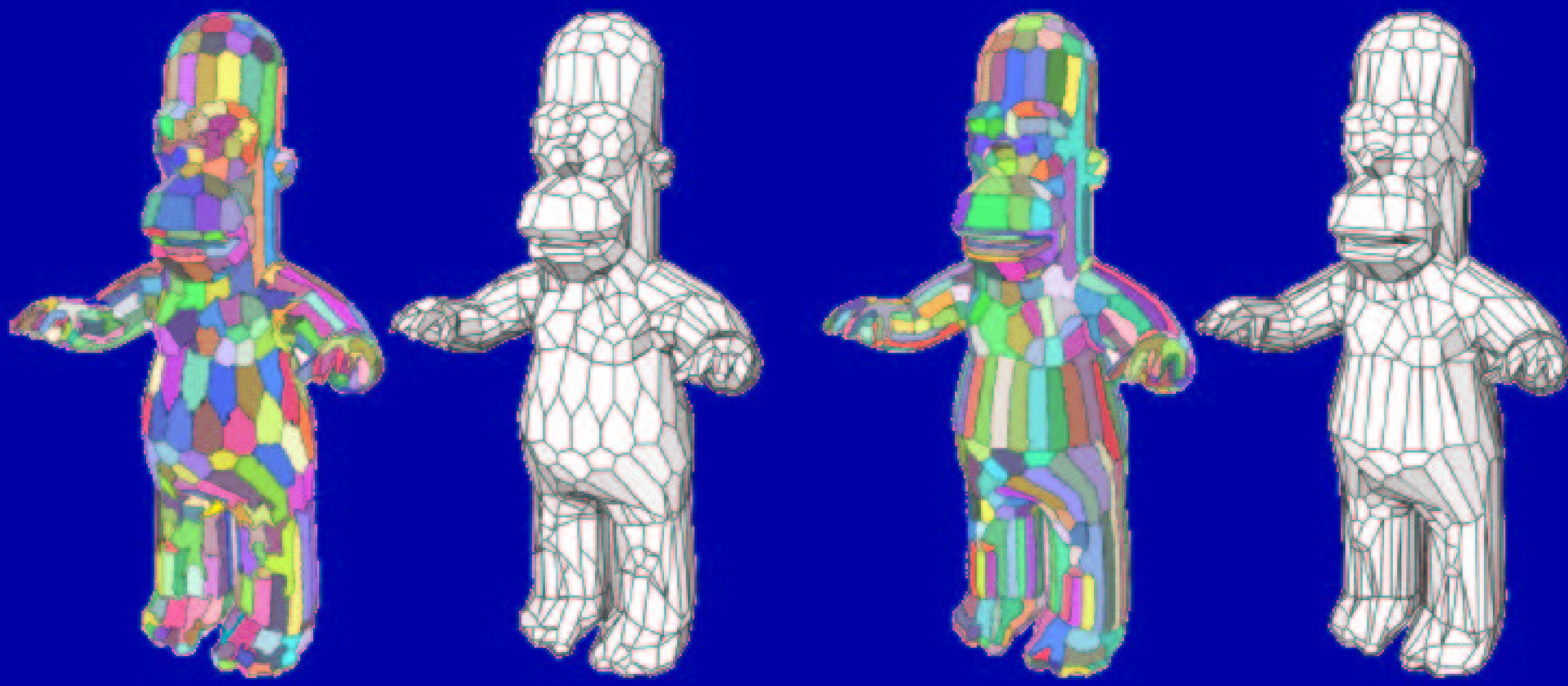




Shape Approximation

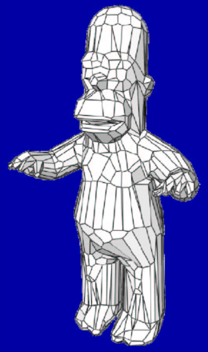
Optimal shape proxies

$$E(R,P) = \sum_{i=1}^{k} \mathbf{d}(R_i, P_i)$$



Optimizing Shape Proxies

Optimizing Shape Proxies

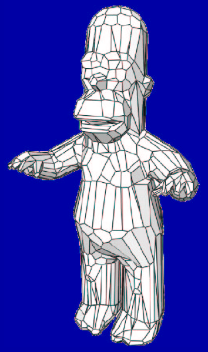


The algorithm

It is inspired in Lloyd's Clustering Algorithm

- **Geometry Partitioning**.
  It is used an error-minimizing region-growing algorithm. The object is segmented in non-overlapping regions.
- **Proxy Fitting**.
  For each partition it is computed an optimal local representative.

Optimizing Shape Proxies

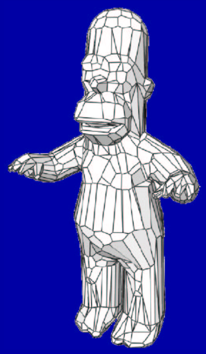


Geometry Partitioning: Initial seed

Given a partition $\mathcal{R}$, a set of proxies $\mathcal{P}$, and an error metric E:

- For each region $\mathcal{R}_i$, select the triangle T_i of $\mathcal{R}_i$ that is most similar to P_i.
- In the first step, select k arbitrary triangles. For each one, assign as proxy its barycenter and normal.

Optimizing Shape Proxies

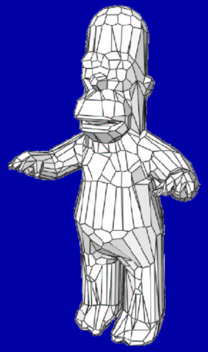


Geometry Partitioning: Region growing

Use a priority queue to "propagate" the regions:

- For each seed triangle T_i, insert its 3 neighbors in the queue
  - Priority = $E(T_j, P_i)$
  - Tag them with the same label of T_i
- For each popped triangle:
  - **IF** it has been assigned to a proxy, **END**
  - **ELSE** Assign it to the region with its tag and push its (untagged) neighbors in the queue

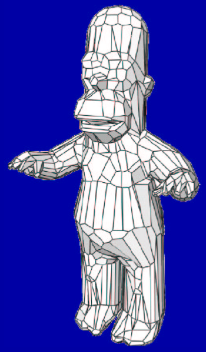
Optimizing Shape Proxies



Proxy Fitting

- $\mathcal{L}_2$ Metric:
  - $\mathbf{x}_i$ = Barycenter of the region $\mathcal{R}_i$.
  - $\mathbf{n}_i$ = Eigenvector of the smallest eigenvalue of the covariance matrix of $\mathcal{R}_i$.
- $\mathcal{L}_{2,1}$ Metric:
  - $\mathbf{x}_i$ = Barycenter of the region $\mathcal{R}_i$.
  - $\mathbf{n}_i$ = area-weighted average of normals of triangles in $\mathcal{R}_i$.

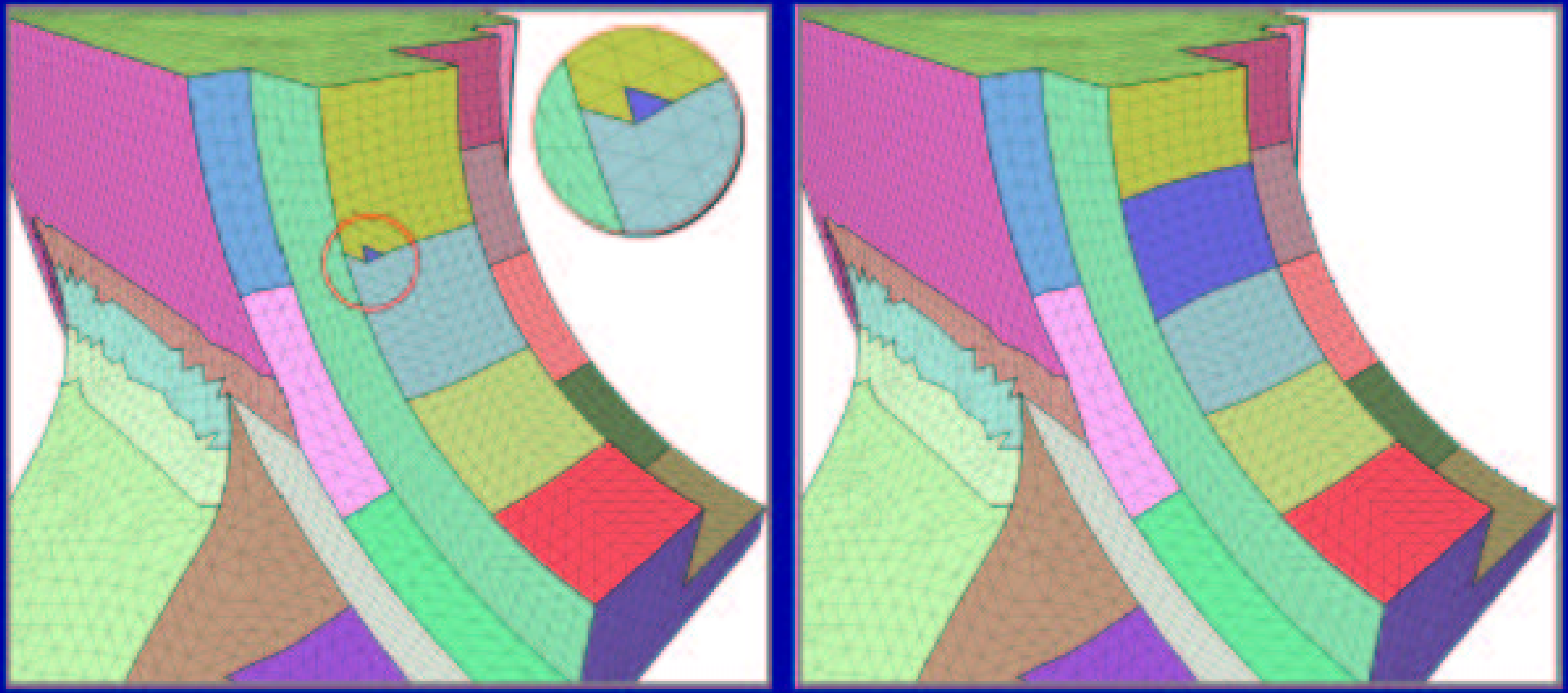
Optimizing Shape Proxies

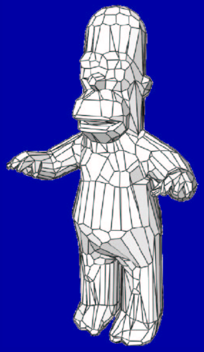


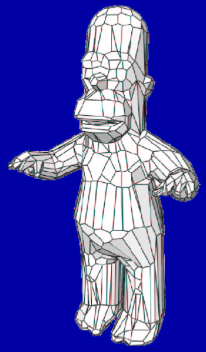
Improvements

- Choosing the number of proxies
- Region Teleportation
- Farthest-point initialization
- Tailoring Refinements
- Smoothing the normal field

Optimizing Shape Proxies







Final remarks and results

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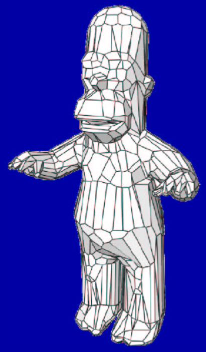
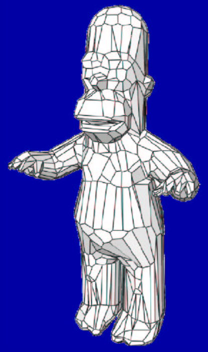


Application to meshing

- Anchor Vertices
- Edge Extraction
- Triangulation
- Polygons

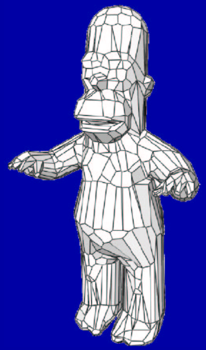
Final remarks and results

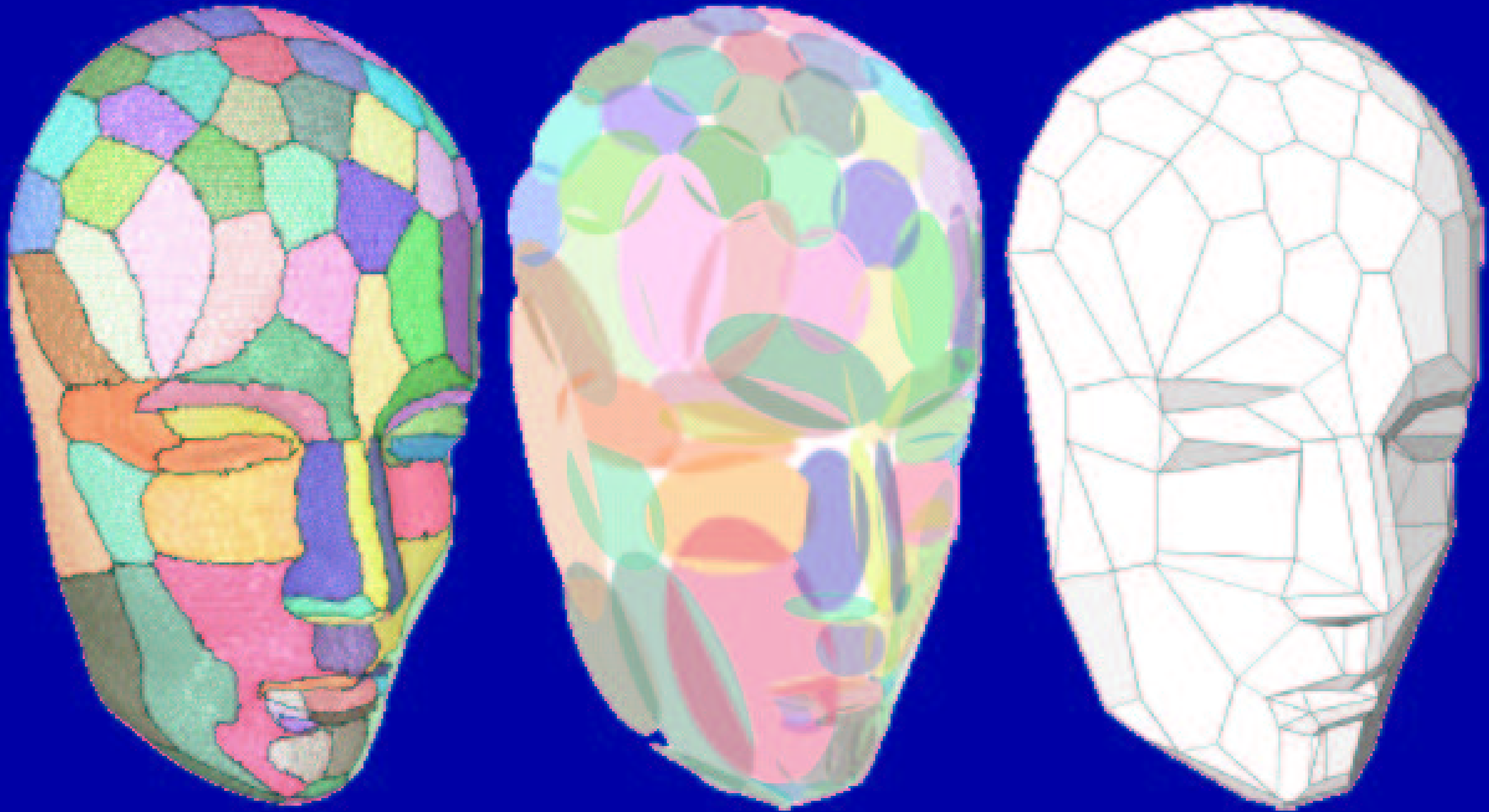


Limitations

- Cannot compete (in time) with greedy methods
- Not Real-Time. I mean, just for offline computations
- Meshing does not handle non-manifold meshes.

Final remarks and results





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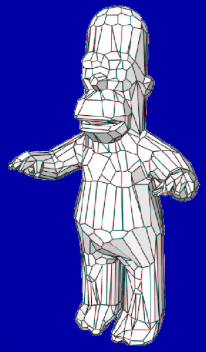

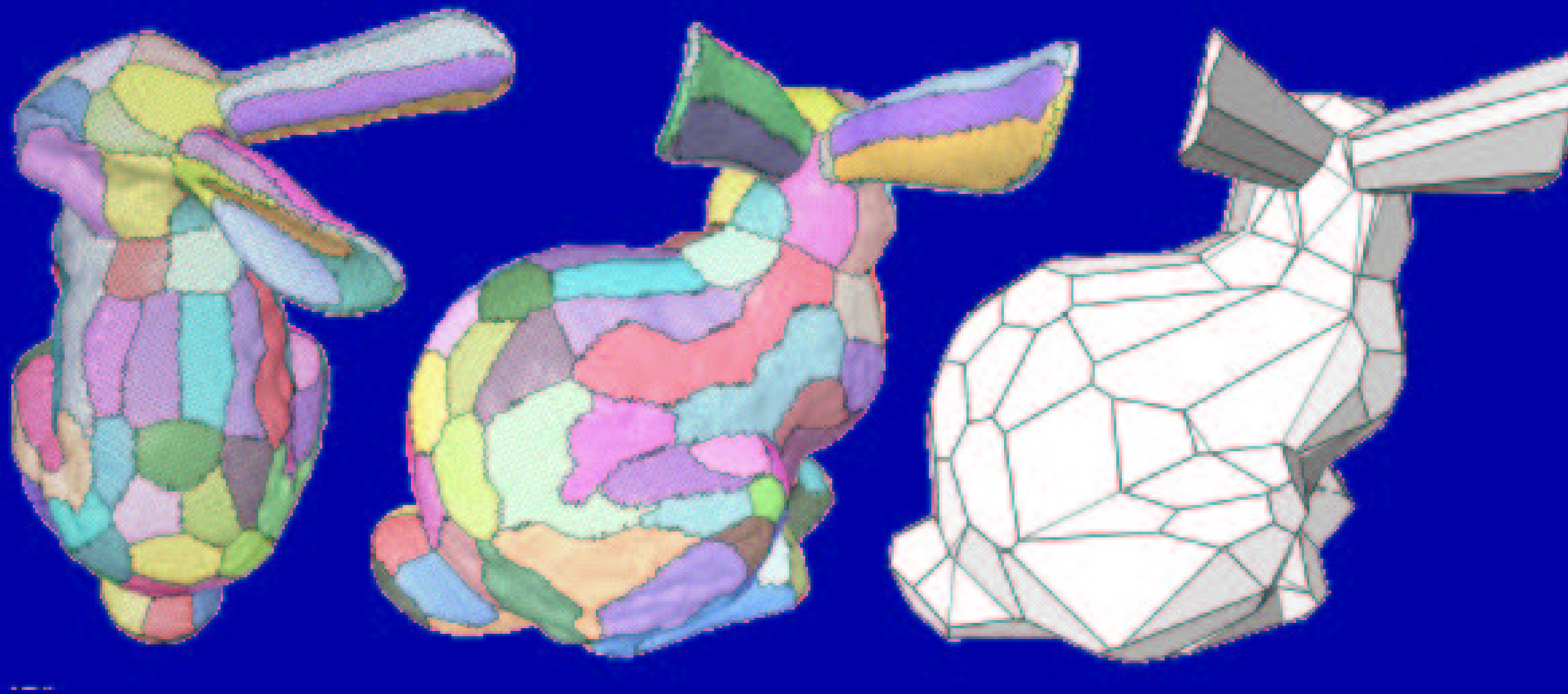

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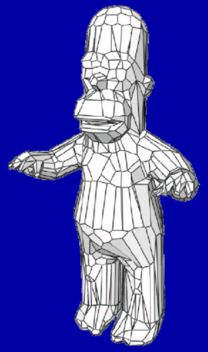

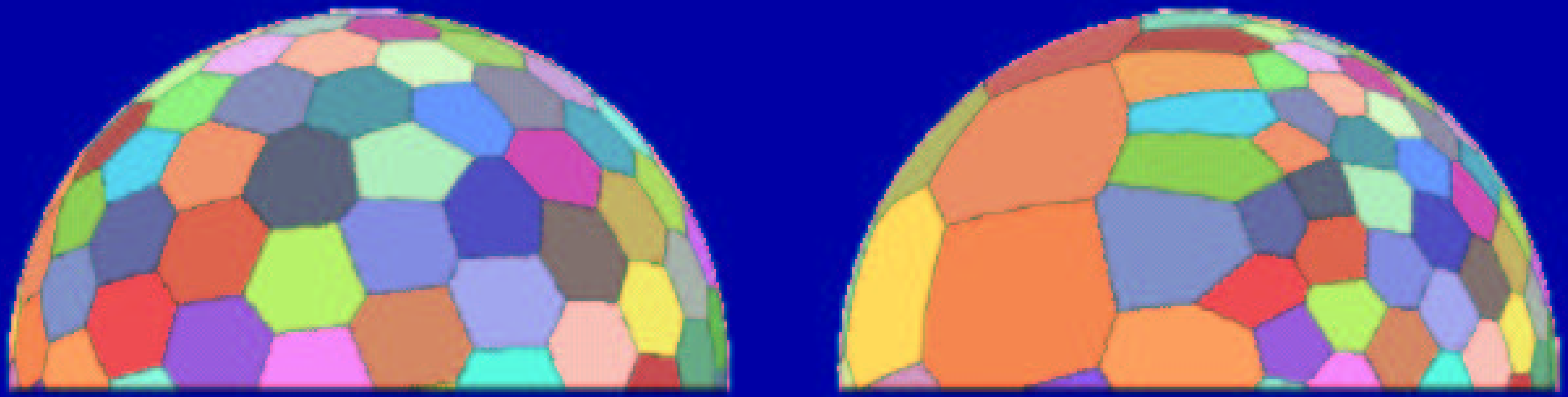

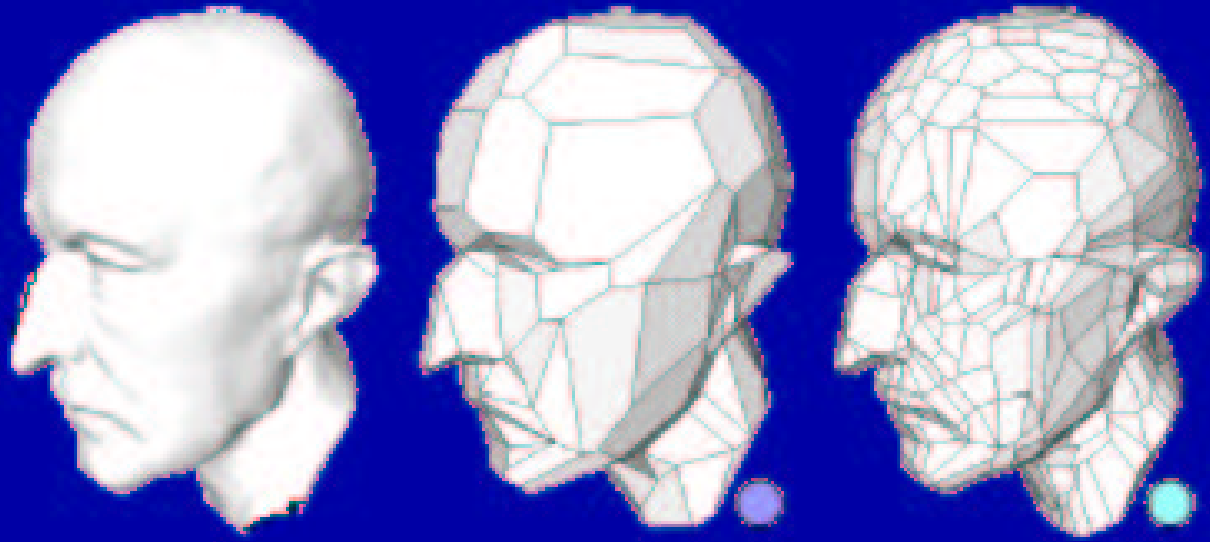

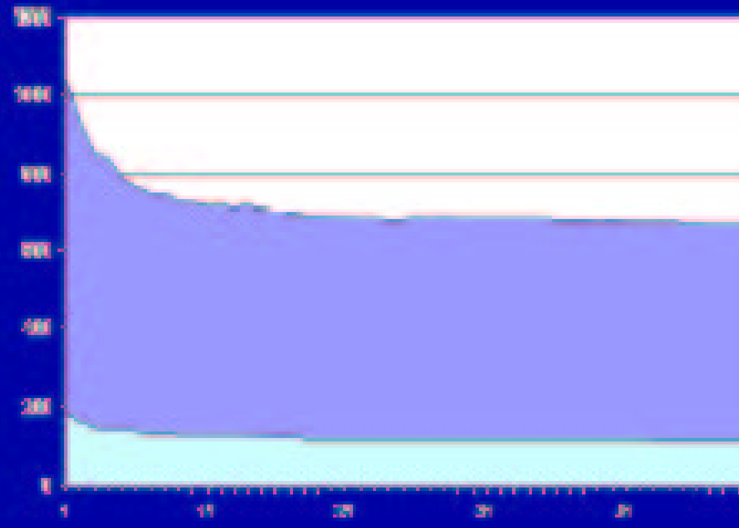

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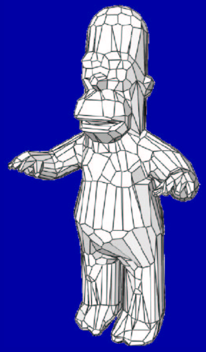

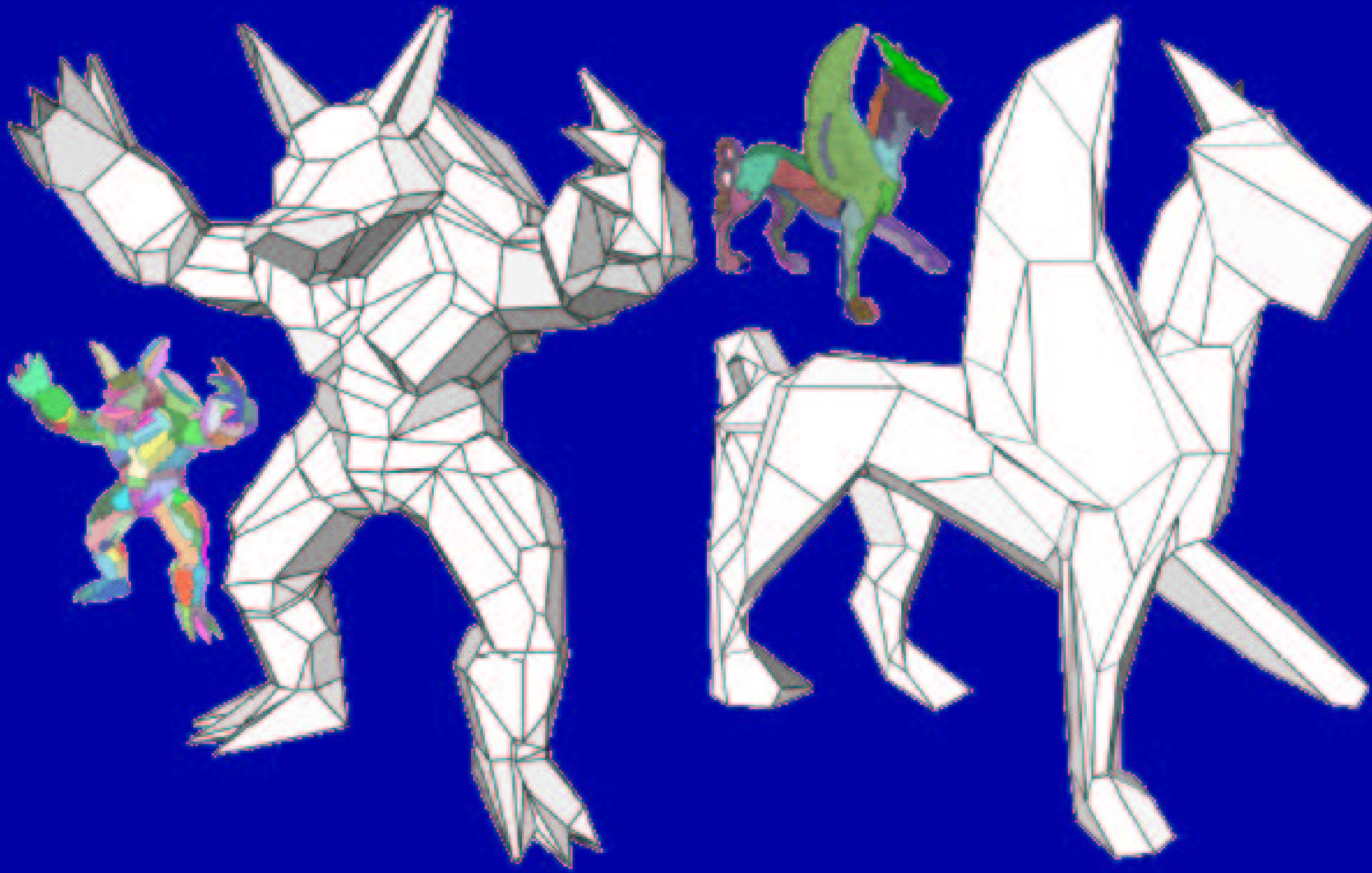

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