PROBLEMS

(Answers to problems marked with * appear at the end of the text.)

2.1 Demonstrate the validity of the following identities by means of truth tables:
(a) DeMorgan's theorem for three variables: $(x + y + z)' = x'y'z'$ and $(xyz)' = x' + y' + z'$
(b) The distributive law: $x + yz = (x + y)(x + z)$
(c) The distributive law: $x(y + z) = xy + xz$
(d) The associative law: $x + (y + z) = (x + y) + z$
(e) The associative law and $x(yz) = (xy)z$

2.2 Simplify the following Boolean expressions to a minimum number of literals:
(a)* $xy + xy'$
(b)* $(x + y)(x + y')$
(c)* $xyz + x'y + xyz'$
(d)* $(A + B)'(A' + B')'$
(e) $(a + b + c')(a'b' + c)$
(f) $a'bc + abc' + abc + a'bc'$

2.3 Simplify the following Boolean expressions to a minimum number of literals:
(a)* $ABC + A'B + ABC'$
(b)* $x'yz + xz$
(c)* $(x + y)'(x' + y')$
(d)* $xy + x(wz + wz')$
(e)* $(BC' + A'D)(AB' + CD')$
(f) $(a' + c')(a + b' + c')$

2.4 Reduce the following Boolean expressions to the indicated number of literals:
(a)* $A'C' + ABC + AC'$ to three literals
(b)* $(x'y' + z)' + z + xy + wz$ to three literals
(c)* $A'B(D' + C'D) + B(A + A'CD)$ to one literal
(d)* $(A' + C)(A' + C')(A + B + C'D)$ to four literals
(e) $ABC'D + A'BD + ABCD$ to two literals

2.5 Draw logic diagrams of the circuits that implement the original and simplified expressions in Problem 2.2.

2.6 Draw logic diagrams of the circuits that implement the original and simplified expressions in Problem 2.3.

2.7 Draw logic diagrams of the circuits that implement the original and simplified expressions in Problem 2.4.

2.8 Find the complement of $F = wx + yz$; then show that $FF' = 0$ and $F + F' = 1$.

2.9 Find the complement of the following expressions:
(a)* $xy' + x'y$
(b) $(a + c)(a + b')(a' + b + c')$
(c) $z + z'(v'w + xy)$

2.10 Given the Boolean functions F_1 and F_2, show that
(a) The Boolean function $E = F_1 + F_2$ contains the sum of the minterms of F_1 and F_2.
(b) The Boolean function $G = F_1F_2$ contains only the minterms that are common to F_1 and F_2.

2.11 List the truth table of the function:
(a)* $F = xy + xy' + y'z$
(b) $F = bc + a'c'$

2.12 We can perform logical operations on strings of bits by considering each pair of corresponding bits separately (called bitwise operation). Given two eight-bit strings $A = 10110001$ and $B = 10101100$, evaluate the eight-bit result after the following logical operations:
(a)* AND (b) OR (c)* XOR (d)* NOT A (e) NOT B

2.13 Draw logic diagrams to implement the following Boolean expressions:
(a) $y = [(u + x')(y' + z)]$
(b) $y = (u \oplus y)' + x$
(c) $y = (u' + x')(y + z')$
(d) $y = u(x \oplus z) + y'$
(e) $y = u + yz + uxy$
(f) $y = u + x + x'(u + y')$

2.14 Implement the Boolean function

$$F = xy + x'y' + y'z$$

(a) With AND, OR, and inverter gates
(b)* With OR and inverter gates
(c) With AND and inverter gates
(d) With NAND and inverter gates
(e) With NOR and inverter gates

2.15* Simplify the following Boolean functions T_1 and T_2 to a minimum number of literals:

A	*B*	*C*	T_1	T_2
0	0	0	1	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	0	1

2.16 The logical sum of all minterms of a Boolean function of n variables is 1.
(a) Prove the previous statement for $n = 3$.
(b) Suggest a procedure for a general proof.

2.17 Obtain the truth table of the following functions, and express each function in sum-of-minterms and product-of-maxterms form:
(a)* $(b + cd)(c + bd)$
(b) $(cd + b'c + bd')(b + d)$
(c) $(c' + d)(b + c')$
(d) $bd' + acd' + ab'c + a'c'$

2.18 For the Boolean function

$$F = xy'z + x'y'z + w'xy + wx'y + wxy$$

(a) Obtain the truth table of F.
(b) Draw the logic diagram, using the original Boolean expression.
(c)* Use Boolean algebra to simplify the function to a minimum number of literals.
(d) Obtain the truth table of the function from the simplified expression and show that it is the same as the one in part (a).
(e) Draw the logic diagram from the simplified expression, and compare the total number of gates with the diagram of part (b).

2.19* Express the following function as a sum of minterms and as a product of maxterms:

$$F(A, B, C, D) = B'D + A'D + BD$$

2.20 Express the complement of the following functions in sum-of-minterms form:
(a) $F(A,B,C,D) = \sum(2, 4, 7, 10, 12, 14)$
(b) $F(x, y, z) = \prod(3, 5, 7)$

2.21 Convert each of the following to the other canonical form:
(a) $F(x, y, z) = \sum(1, 3, 5)$
(b) $F(A, B, C, D) = \prod(3, 5, 8, 11)$

2.22* Convert each of the following expressions into sum of products and product of sums:
(a) $(u + xw)(x + u'v)$
(b) $x' + x(x + y')(y + z')$

2.23 Draw the logic diagram corresponding to the following Boolean expressions without simplifying them:
(a) $BC' + AB + ACD$
(b) $(A + B)(C + D)(A' + B + D)$
(c) $(AB + A'B')(CD' + C'D)$
(d) $A + CD + (A + D')(C' + D)$

2.24 Show that the dual of the exclusive-OR is equal to its complement.

2.25 By substituting the Boolean expression equivalent of the binary operations as defined in Table 2.8, show the following:
(a) The inhibition operation is neither commutative nor associative.
(b) The exclusive-OR operation is commutative and associative.

2.26 Show that a positive logic NAND gate is a negative logic NOR gate and vice versa.

2.27 Write the Boolean equations and draw the logic diagram of the circuit whose outputs are defined by the following truth table:

Table P2.27

f_1	f_2	a	b	c
1	1	0	0	0
0	1	0	0	1
1	0	0	1	0
1	1	0	1	1
1	0	1	0	0
0	1	1	0	1
1	0	1	1	1

2.28 Write Boolean expressions and construct the truth tables describing the outputs of the circuits described by the logic diagrams in Fig. P2.28.

2.29 Determine whether the following Boolean equation is true or false.

$$x'y' + x'z + x'z' = x'z' + y'z' + x'z$$

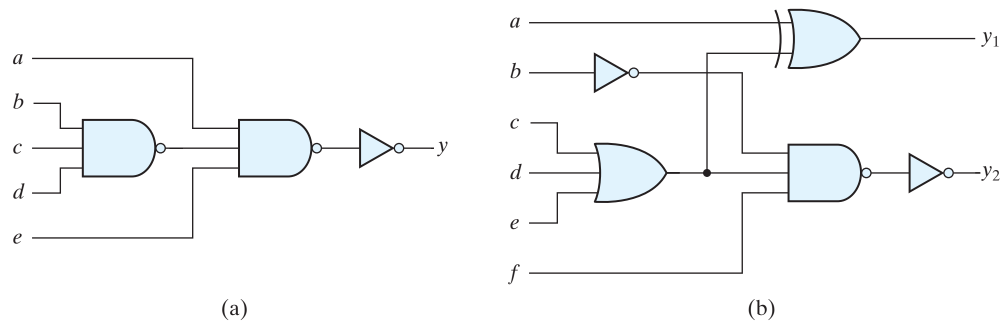

FIGURE P2.28

2.30 Write the following Boolean expressions in sum of products form:

$$(b + d)(a' + b' + c)$$

2.31 Write the following Boolean expression in product of sums form:

$$a'b + a'c' + abc$$

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WEB SEARCH TOPICS

Algebraic field
Boolean logic
Boolean gates
Bipolar transistor
Field-effect transistor
Emitter-coupled logic
TTL logic
CMOS logic
CMOS process