## © Copyright 2024 Saad Mneimneh It's illegal to upload this document on any third party website CSCI 705 Algorithms Homework 3

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## Readings

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Based on Lectures 5, 6 and their assigned readings (see course website).

Problem 1: Looking into Quicksort

The analysis of randomized Quicksort relied on that all elements are distinct. We assume that the partitioning algorithm is the one presented in class, where A[r] is the pivot:

RandPartition(A, p, r)

 $i \leftarrow \operatorname{random}(p, r)$ swap  $A[r] \leftrightarrow A[i]$ return Partition(A, p, r) (see Partition in lecture notes)

(a) Explain why the assumption about elements being distinct is needed.

(b) Suggest a way to change the partitioning algorithm to overcome the problem of the multiplicity of elements.

(c) Assume that elements may not be distinct, but that every element cannot appear more then c times, where c is a constant. How does that affect the asymptotic running time of randomized Quicksort? Assume the elements are  $z_1 \leq z_2 \leq \ldots \leq z_n$ . For a given  $z_i < z_j$ , what is the probability that they are compared (one of them must be the first to become pivot among what set of elements?)



## Problem 2: Nesquiksort

Nesquiksort works exactly like randomized Quicksort, except that after the array is partitioned around a random pivot, the right partition gets sorted in O(1) time by a magical bunny. Let T(n) be the running time of Nesquicksort on an array of size n. As we did with Quicksort, assume that the elements are distinct and belong to  $\{1, 2, \ldots, n\}$ .

(a) Write a recurrence for T(n) similar to the one we had for randomized Quicksort.

(b) Guess a solution for the recurrence and verify it by the substitution method. Show your work.

(c) Analyze Nesquicksort using the indicator random variable technique. Let  $X_{ij}$  be an indicator for the event that i and j are compared. Find  $E[X_{ij}]$ , which is equal to the probability that i and j are compared. *Hint*: For i and j to be compared, one of them must be the first to become the pivot among which set of elements? Once you find  $E[X_{ij}]$ , find the expected number of comparisons.

## Problem 3: matching socks

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There are n pairs of socks. Each pair has a distinct color. However, the 2n socks are randomly permuted, with every permutation being equally likely. To wear socks, we repeatedly try the next sock until we have a matching pair. How many socks do we expect to try? To answer this question, we will go through a series of steps.

(a) Show that the number of permutations that do not have any match within the first i socks is:

$$\frac{n!}{(n-i)!}2^i(2n-i)!$$

and based on that, find the probability that there is no match within the first i socks.

(b) Define the indicator random variable  $X_i$  as follows:

 $X_i = \begin{cases} 1 & \text{if there is no match within the first } i \text{ socks} \\ 0 & \text{otherwise} \end{cases}$ 

Express the number of trials until we get a match in terms of indicator random variables, and find the expected number of trials. You might get a complicated expressions, but that's ok. Try to make it look like:

$$\frac{\sum_{i=0}^{n}\dots}{\binom{2n}{n}}$$

(c) Analyze the result in (c), either experimentally or mathematically to obtain the asymptotic behavior of the number of trials needed to find a matching pair of socks. For instance, try to figure out what the numerator in the above form is for few values of n. In addition, replace the denominator by factorials and use Stirling's approximation.

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