CSCI 705 Algorithms
Homework 4
Due 3/8/2024

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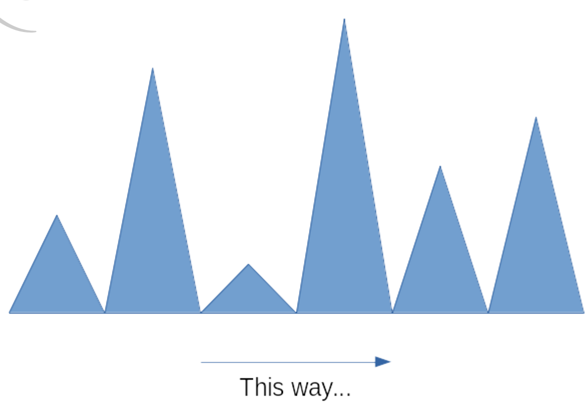
Readings
Based on Lectures 8 and 9 and their assigned readings. This homework focuses on doing things in linear time, or finding that we can't.

Problem 1
Given an array $A[1 \ldots n]$ of distinct integers in the range $\{1, \ldots, n^4\}$, and a target integer t, output all pairs $A[i]$ and $A[j]$ such that $A[i] + A[j] = t$. The ideal solution should have $O(n)$ running time.

Problem 2
We have n people standing on n mountain peaks. The following figure shows an example of these peaks for $n = 6$.



Each person needs to attempt the next challenge, which is to climb the higher mountain closest to their current position and to their right.

The n peaks can be represented by an array of numbers (and let's assume they are integers); for instance, the array for the above example cab be $A = [2, 5, 1, 6, 3, 4]$. The problem becomes as follows: For each index i, find the smallest index j such that $j > i$ and $a[j] > a[i]$. If no such j exists, make $j = i$ (the person stays). For instance, if we imagine another array B, the answer to all the j's for this instance would be $B = [2, 4, 4, 4, 6, 6]$.

(a) Show that the straight forward approach in which we check for each i, every $j = i + 1, i + 2, \ldots, n$, requires $\Omega(n^2)$ time. *Hint*: construct an input that forces the $\Omega(n^2)$ time.

(b) Find an algorithm that performs the task in $O(n)$ time.

Problem 3

Given an array $A[1 \ldots n]$ of numbers, assume that the i^{th} smallest element is guaranteed to lie somewhere between position $i - k$ and $i + k$ for all $i = 1 \ldots n$, where k is some fixed constant.

(a) Design an algorithm to sort the array in $O(n \log k)$ time.

(b) Show that any algorithm that sorts the array requires $\Omega(n \log k)$ time. *Hint*: Given a sorted array A, think about ways to permute A to satisfy the above condition, and use a decision tree argument.

Problem 4

Given an array A that is not sorted, we want to find two elements, call them x and y, that are "close enough". For instance, consider the average distance between consecutive elements in the sorted order. This average distance can be computed as

$$avgD = \frac{(z_2 - z_1) + (z_3 - z_2) + \ldots + (z_n - z_{n-1})}{n - 1}$$

(but it can also be computed without knowing the sorted order)
So let's call two elements x and y close enough if $|x - y| \leq avgD$. Find two such elements in linear time. *Hint 1*: use divide-and-conquer to work with two balanced subproblems. *Hint 2*: What can you say about the average distance in a one of the two subproblems?