Stat 319/739 Bayesian Stat

course website: http://www.cs.hunter.cuny.edu/~saad/courses/bayes

- Make sure you read your cunyfirst email
- At some point I will ask you to sign up with gradescope.com
  (this will be used to submit HW, etc...)

Why Bayesian ?

1) Generalized probabilistic framework

classical approach is special case

2) Formulates interesting questions ?

e.g. classical approach: Ask questions about outcomes of coin

But if we ask something related to property of coin such as is it fair ?

Bayes will help formulate this question mathematically.

3) Treat prob. parameters as random variables themselves.

classical approach: Coin is fair $\Rightarrow P(H) = \frac{1}{2}$

parameter $p = 1/2$

Bayesian: p is unknown.

- Example: There is a fifty percent chance the coin is fair $P(p = \frac{1}{2}) = \frac{1}{2}$

Bayes: prob $\neq$ frequency

- Example: Laplace used Bayesian approach to estimate the mass of saturn.

Mass of saturn in principle is deterministic

error in measuring gives it a prob. aspect (Modeling)

Bayesian Approach $\Longleftrightarrow$ Conditioning

Probability:

Experiment generates outcomes

Set of outcomes (For now discrete, i.e. not continuous)

- $S = \{H, T\}$ tossing coin
- $S = \{1, 2, 3, 4, 5, 6\}$ Rolling die
- S can be infinite

Each outcome $s_i \in S$ has $P(s_i) \geq 0$

Axioms:

(1) • $P(s_i) \geq 0$

(2) • $P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$ $(A_i \cap A_j = \phi)$

(disjoint) (exclusive)

(3) • $P(S) = 1$

Basic version of (2): $P(A \cup B) = P(A) + P(B) \quad A \cap B = \emptyset$

Example: $E = \{s_1, s_2, s_3, \dots\}$

$$P(E) = P(s_1) + P(s_2) + P(s_3) + \dots$$

because $E = \{s_1\} \cup \{s_2\} \cup \{s_3\} \cup \dots$

$$P(S) = \sum P(s_i) = 1$$

$$P(\emptyset) = 0$$

The set of outcomes together with the prob. gives us what we call prob. space.

<u>Example:</u> Uniform prob. space $S = \{s_1, s_2, \dots, s_k\}$

$$P(s_i) = 1/k$$

$$A \text{ is an event} \iff A \subset S$$

$$P(A) = \sum_{s \in A} P(s)$$

Conditioning

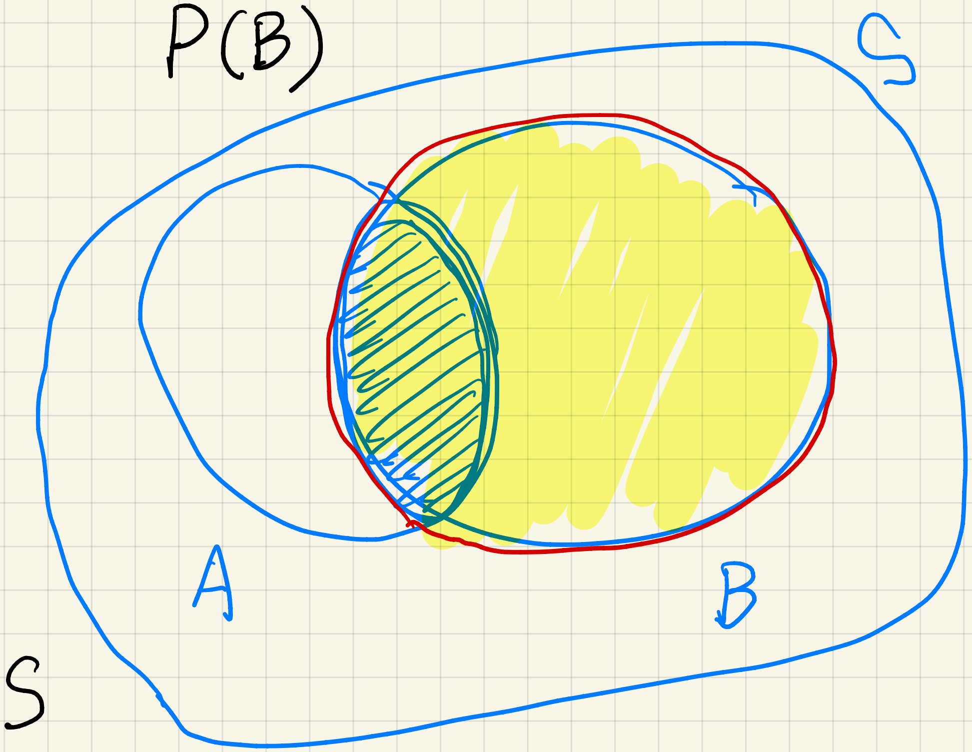
$$\underbrace{P(A|B)}_{\text{Prob. of } A \text{ conditioned on } B} = \frac{P(A,B)}{P(B)}$$

"and"

$$P(\underbrace{A \cap B}_{\text{set}})$$

Prob of A given B

$P(B) \neq 0$



B is the "new world" not S

Special Case: $B = S$

$$P(A|S) = \frac{P(A,S)}{P(S)} = \frac{P(A\cap S)}{1} = \frac{P(A)}{1}$$

$$P(A|S) = P(A)$$

Independence

$S = \{1,2,3,4,5,6,7,8\}$ $P(s_i) = \frac{1}{8}$

- A: 1 2 3 4 B: 5, 6, 7, 8

$P(A) = 1/2$ $P(B) = 1/2$

$P(A\cap B) = P(\phi) = 0 \neq 1/2$

X

- A: 1 2 3 4 B: 3 4 5 6 7 8

$P(A) = 1/2$ $P(B) = 1/2$

$$P(A\cap B) = P(\{3,4\}) = 1/4 = \frac{1}{2}\times\frac{1}{2}$$

✓ independent

- 1 2 3 4 5 6 7 8

$P(A) = 1/2$

$P(B) = 1/2$

$$P(A\cap B) = P(\{2,3,4\}) = \frac{3}{8} \neq \frac{1}{2}\times\frac{1}{2}$$

X

A and B are independent iff

- $P(A \cap B) = P(A) \cdot P(B)$

- $\underline{\underline{P(A|B)}} = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A) \cdot \cancel{P(B)}}{\cancel{P(B)}} = \underline{\underline{P(A)}}$

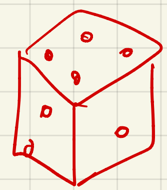
- $P(B|A) = P(B)$

Example:

$A = \{2, 4, 6\}$

$S = \{1, 2, 3, 4, 5, 6\}$

$B = \{3, 6\}$



Uniform Prob. Space

$C = \{1, 2, 3, 4, 5\}$

$P(A) = 1/2 \qquad P(B) = 1/3 \qquad P(C) = 5/6$

$P(A, B) = 1/6 = P(A) \cdot P(B)$ (A & B are independent)

A : event that we get even number

B : event that we get a multiple of 3

$P(A | B) = P(A) = \frac{1}{2}$

Given that we rolled a multiple of 3, the prob. that it's even is still 1/2