Problem 0: Readings
Read note 4 and 5 from the course web site.

Problem 1: Accidents
Assume there is a dangerous intersection where accidents occur randomly in time and the number of accidents during an interval of $t$ days is assumed to be Poisson distributed with mean $t\lambda$. The parameter $\lambda$ is the daily accident rate. The possible values for $\lambda$ are:

\{0.5, 1, 1.5, 2, 2.5, 4\}

with respective probabilities

\{0.1, 0.2, 0.3, 0.2, 0.15, 0.05\}

(a) Given that we observe 12 accidents in a six-day period, find the posterior probability distribution for $\lambda$. You may use a programming language as helper to avoid manual calculations, e.g. R or excel.

(b) What is the probability that there will be no accidents during the next week?

Problem 2: Poisson approximation
Assume that the production of items follows a Bernoulli scheme of trials, and the probability that an item is defective is $p = 0.015$. Consider boxes that contain $n = 100$ items.

(a) Using Poisson approximation, find approximate value for the probability of finding $k$ defective items. Compare to exact probabilities for small values of $k$, say $k = 0$, $k = 1$, and $k = 2$.

(b) Assume that we are not sure about the value of $p$, but it could be any value in the following set:

\{0.005, 0.015, 0.025\}

We open a box and find 2 defective items. What should we believe the value of $p$ is? Explain your answer.
Problem 3: Central Limit
Consider tossing a fair coin $100n$ times, where $n$ is geometrically distributed with mean 6. Due to a counting error, it is reported that the fraction of heads is in $[0.5, 0.51]$. What is the most likely value for $n$? In other words, we are looking for the value of $k$ that maximizes $P(n = k | 0.5 < \frac{S_{100n}}{100n} < 0.51)$, where $S_n$ is the number of heads obtained in $n$ coin tosses.