Problem 0: Readings
Read note 6 (partially) on the course web page.

Problem 1: Normal observations
Suppose that you are given 12 observations from a normal distribution:

\[ \begin{align*}
15.644 & \quad 16.437 & \quad 17.287 & \quad 14.448 \\
15.308 & \quad 15.169 & \quad 18.123 & \quad 17.635 \\
17.259 & \quad 16.311 & \quad 15.390 & \quad 17.252
\end{align*} \]

and we are told that the variance \( \sigma^2 = 1 \). Find the 90\% high density region for the posterior distribution of the mean using a suitable prior of your choice. The 90\% high density region is defined as the region that leaves 5\% probability on each side.

Problem 2: Two groups
Consider the following two groups:

\[ \begin{align*}
\bar{x} & = 216 & \bar{y} & = 210 \\
\sigma^2_x & = 2210 & \sigma^2_y & = 2618 \\
n & = 130 & m & = 119
\end{align*} \]

where given \( \mu_x, x_1, \ldots, x_n \) are independent and indentically distributed normal random variables with mean \( \mu_x \) and variance \( \sigma^2_x \) (which is known). Same for the \( y \)'s.

(a) Use a P-value approach to decide whether \( \mu_x = \mu_y \).

(b) Use a Bayesian approach with an improper uniform prior to find \( P(\mu_x - \mu_y > b|\bar{x}, \bar{y}) \) for \( b = 0, 1, 2, \ldots, 10 \). Plot the curve \( P(\mu_x - \mu_y > b|\bar{x}, \bar{y}) \) vs. \( b \) to have a visual representation.

(c) Compare both approaches.

(d) Let us now assume that \( \mu_x \sim \mu_y \sim N(\beta, \tau^2) \) as we did in class. Find the posterior density \( f(\mu_x - \mu_y|x, y) \). Note this is not the same as \( f(\mu_x - \mu_y|x, y) \). The advantage of this alternate conditioning is that it removes the dependence on the parameter \( \beta \) (why?) and simplifies the posterior. Repeat part (b) for
two cases $\tau^2 = \sigma^2$ and $\tau^2 = 2\sigma^2$, where $\sigma^2 = \sigma_x^2/n + \sigma_y^2/m$. Hint on how to approach this: let $w = \mu_x - \mu_y$ and $d = \bar{x} - \bar{y}$ and observe that given $w$, $d$ is normal with mean $w$. 