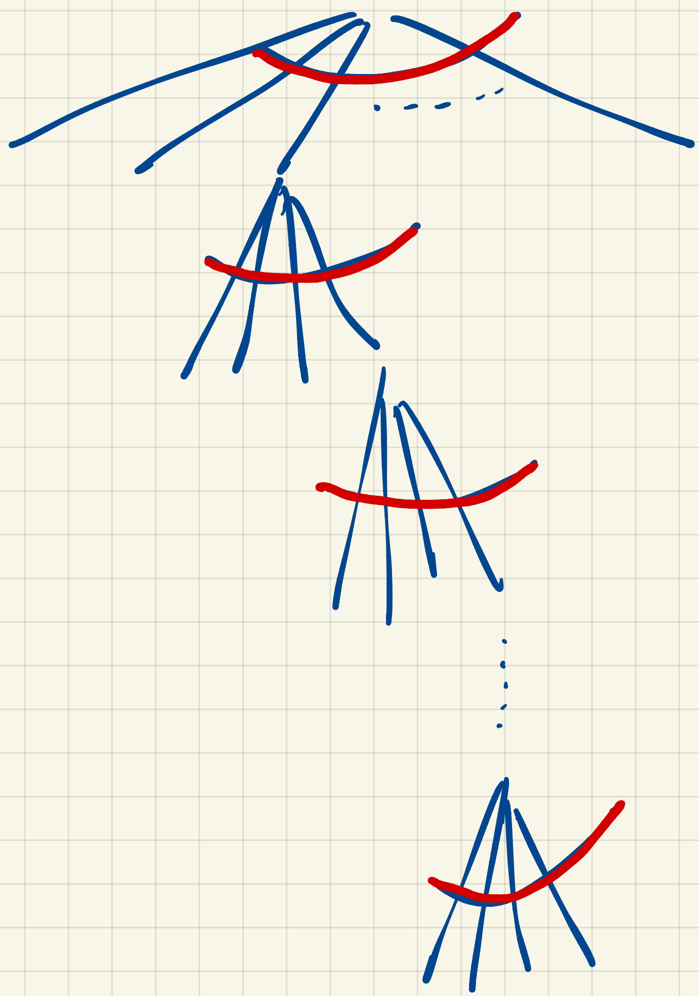
ordered selection of k out of n

k-permutations



phase	
1.	n
2.	$n-1$
3.	$n-2$
$\vdots$	$\vdots$
k.	$n-k+1$

$$n(n-1)(n-2)\dots(n-k+1) = \frac{n!}{(n-k)!}$$

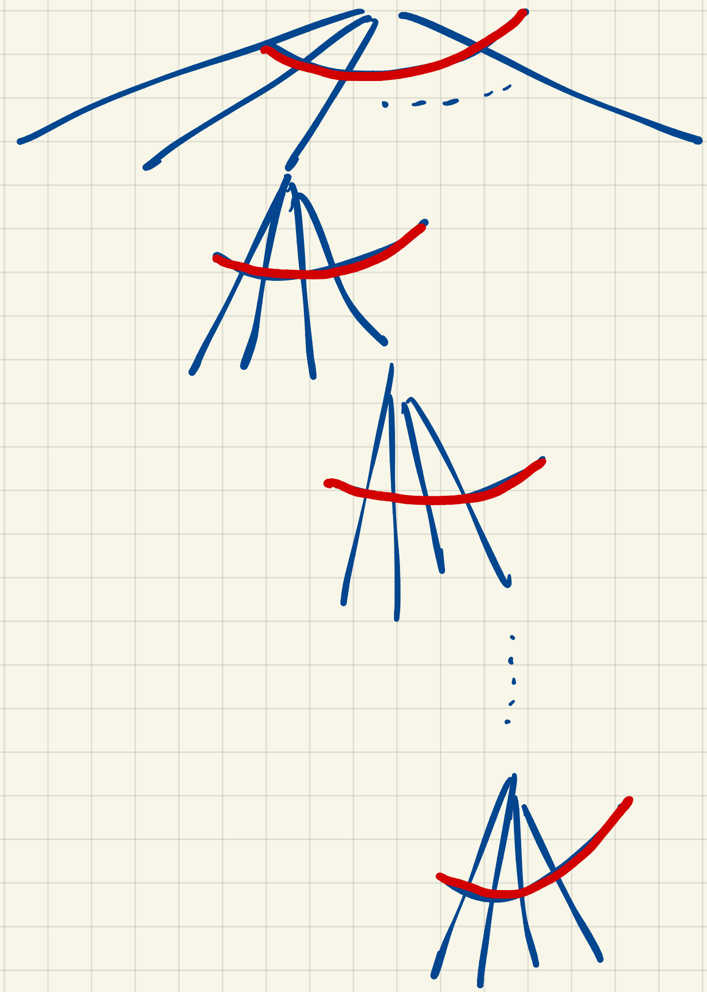
unordered selection of k out of n

\# combinations

- In this case, every permutation of the k choices in the above process produces an equivalent outcome
- The above process overcounts by $k!$
- $$\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$$
- $\binom{n}{k}$ is the number of subsets of $S = \{1, 2, 3, \dots, n\}$ of size k. For example, if $S = \{1,2,3,4,5\}$, there are $\binom{5}{3} = 10$ subsets of S of size 3.

$\{1,2,3\}$ $\{1,2,4\}$ $\{1,2,5\}$ $\{1,3,4\}$ $\{1,3,5\}$ $\{1,4,5\}$ $\{2,3,4\}$ $\{2,3,5\}$ $\{2,4,5\}$ $\{3,4,5\}$

ordered selection of k out of n with repetition

phase		
1.		n
2.		n
3.		n
$\vdots$		
k.		n

$$\underbrace{n \times n \times \dots \times n}_{k} = n^k$$

	repetition	no repetition
order	n^k	$\frac{n!}{(n-k)!}$
no order	?	$\binom{n}{k}$

Later

Examples:

- Given n movies, select the best k : $\binom{n}{k}$
- Same, but rank them : $\frac{n!}{(n-k)!}$
- Given k movie nights and n movies, select what to watch : n^k

Be careful about "order"

- ordered selection v.s. unordered selection must be chosen as a model rather than being <u>literal</u>
- Example:
  - choose a sequence of 3 letters from the alphabet (no repetition) such that letters appear in alphabetical order
  - choose a sequence of 3 letters from the alphabet (no repetition) in any order

The first is $\binom{26}{3}$ and the second is $\frac{26!}{(26-3)!}$ despite the wording!

$\binom{n}{k}$ as a special case of anagrams.

- Consider permuting the letters in MISSISSIPPI
This can be done in $11!$ ways (11 letters). But for each anagram obtained, permuting all I's or all S's or all P's would still produce the same anagram.
So to account for over counting, we have $\dfrac{11!}{4!\,4!\,2!}$
- In general, if we have k letters, and letter i occurs n_i times, the # anagrams is $\dfrac{n!}{n_1!\,n_2!\cdots n_k!}$, $n = \sum_{i=1}^{k} n_i$
- $\binom{n}{k}$ can be thought of as number of anagrams with k 1's and $(n-k)$ 0's, so $\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$