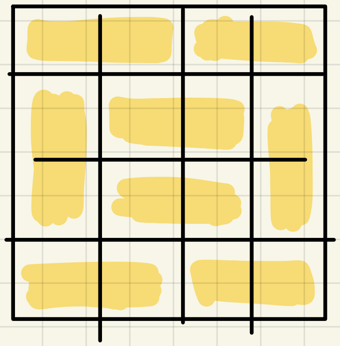
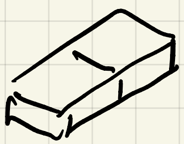
More proofs

Consider a $n \times n$ board. (n is even)

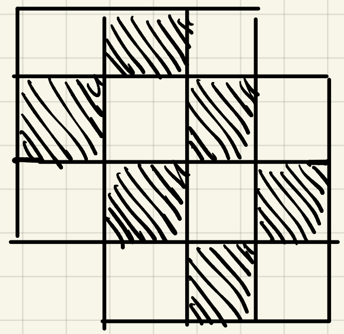


(Example $n=4$)

Dominos:



Can we cover the board with Dominos?



If we delete 2 opposite corners can we still cover the board with Dominos?

P: Two opposite corners are deleted

Q: Board is <u>not</u> coverable

$(P \Rightarrow Q)$ is true.

<u>Proof by contradiction:</u>

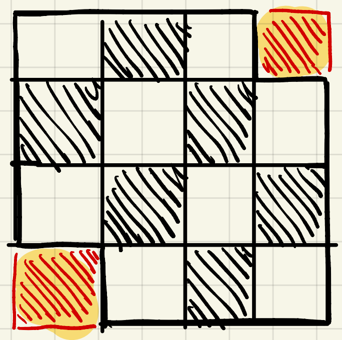
How would I start:

Start with $P \wedge \neg Q$ (that's the negation of $P \Rightarrow Q$)

<u>Parity argument:</u> make every square even or odd

2 adjacent squares have different parity.

e.g. Square is even if sum of its coordinates is even



n even means that

\# even square = # odd squares

P : Two opposite corners deleted

$\Rightarrow$ two square of the same parity deleted

$\Rightarrow$ # even square $\neq$ # odd squares

$\neg Q$: board is Coverable

$\Rightarrow$ every Domino covers two adjacent square with diferent parity

$\Rightarrow$ # even square = # odd square

$P \wedge \neg Q \Rightarrow$ False. Therefore $(P \Rightarrow Q)$ is true

Prove by contradiction that primes are infinite.

Prime are finite $\Rightarrow$ the set of prime numbers is finite

$\Rightarrow$ say it's $\mathbb{P} = \{ p_1, p_2, p_3, \ldots, p_k \}$ where $k \in \mathbb{N}$

Magic (Light bulb): $n = 1 + \prod_{i=1}^{k} p_i$, $n \in \mathbb{N}$

$\Rightarrow$ n is not divisible by any of the prime numbers (division has remainder 1)

a contradiction since every integer can be factored into primes.

Generalizing even/odd results

Prove: (1). n even $\Longrightarrow$ n^k even , $k \in \mathbb{N}$

(2). n odd $\Longrightarrow$ n^k odd , $k \in \mathbb{N} \cup \{0\}$

(1): n is even $\Rightarrow$ $n = 2m$, $m \in \mathbb{Z}$

$$\Rightarrow n^k = (2m)^k = 2^k \cdot m^k = 2\underbrace{(2^{k-1} \cdot m^k)}_{\in \mathbb{Z}\,?}$$

Yes

$$= 2 \cdot m' , \; m' \in \mathbb{Z}$$

$\Rightarrow n^k$ is even.

(2) : n is odd $\Rightarrow n = (2m+1)$, $m \in \mathbb{Z}$

$$\Rightarrow n^k = (2m+1)^k$$

$$= \underbrace{\binom{k}{0}(2m)^k + \binom{k}{1}(2m)^{k-1} + \cdots \binom{k}{k-1}(2m)^1}_{2\left[\underbrace{\binom{k}{0}2^{k-1}m^k + \cdots\cdots + \binom{k}{k-1}m}_{\in \mathbb{Z}}\right]} + \underbrace{\binom{k}{k}(2m)^0}_{+\ 1}$$

$\Rightarrow n^k$ is odd

Prove: There is no smallest positive rational number.

Let $x > 0$ be the smallest positive rational number

let $y = \frac{x}{2}$ ($y > 0$, $y \in \mathbb{Q}$ because if $x = \frac{a}{b}$ then $y = \frac{a}{2b}$)

contradiction since $y < x$.

$$\text{Prove}: \quad \overbrace{n \text{ odd}}^{P} \Longrightarrow \overbrace{n = a^2 - b^2 \quad \text{where } a, b \in \mathbb{Z}}^{Q}$$

$$n \text{ is odd} \Longrightarrow n = 2k + 1 \quad , k \in \mathbb{Z}$$

$$= \underbrace{(k+1)^2}_{k^2+2k+1} - k^2$$

Contrapositive: $n \neq a^2 - b^2 \Longrightarrow n$ is even.

The contrapositive is not very helpful here.

It's easier to express "$=$" than "$\neq$"