$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

start with 1

jump by 1

end with n

Generalize

start with a

jump by s

end with b

$$a + (a+s) + (a+2s) + \dots + b \overset{?}{=} \left(\frac{b-a}{s}+1\right)\left(\frac{a+b}{2}\right)$$

$$\#\text{terms} \times \text{avg}(a,b) = \#\text{terms}\left(\frac{a+b}{2}\right)$$

$$= \left(\frac{b-a}{s}+1\right)\left(\frac{a+b}{2}\right)$$

$$\begin{array}{ccccccc} 1 & 2 & 3 & 4 & \cdots & (n-1) & n \\ n & (n-1) & (n-2) & \cdots & & 2 & 1 \\ \hline \end{array}$$

$$\underbrace{(n+1) \quad (n+1) \quad \cdots \quad (n+1)}_{n}$$

total: $n(n+1)$

$$1+2+3+\cdots+n = \frac{n(n+1)}{2} = \#\text{terms} \cdot \text{Avg}(1,n)$$

What about?

$$1 \times 2 \times 3 \times \cdots \times n = n!$$

(n factorial or factorial of n)

$1! = 1$

$2! = 2$

$3! = 6$

$4! = 24$

## What does $n!$ represent?

This is the # of permutations on $n$ things.

Example: there are $10!$ ways of stacking 10 books

Lazy professor: - Professor does not want to grade

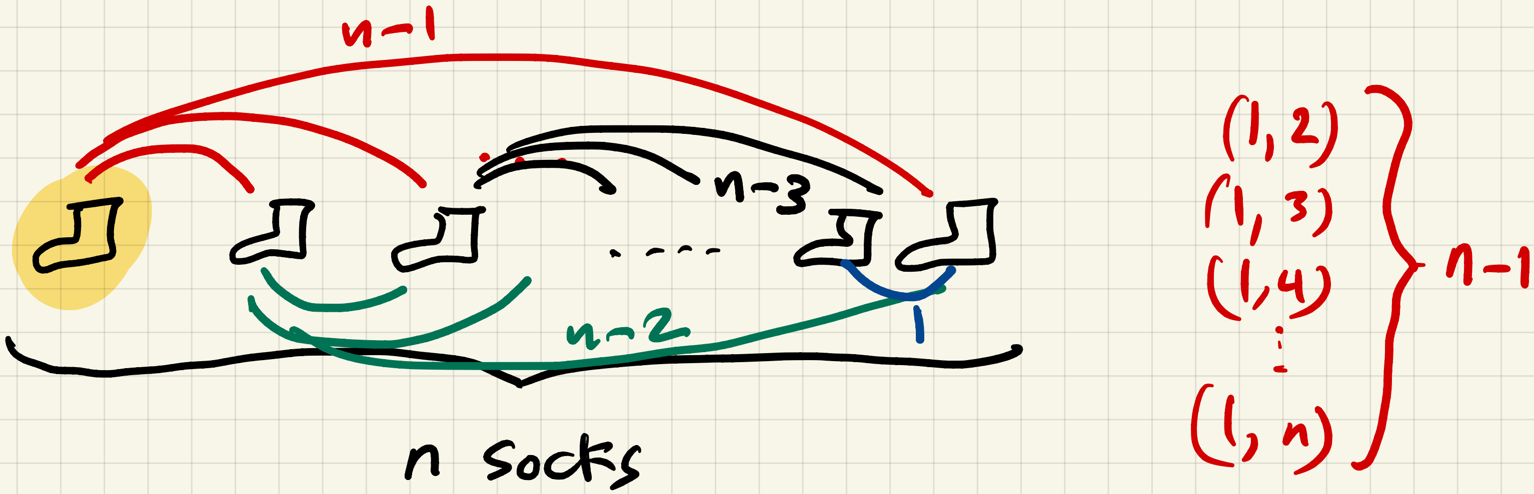
- Permute the tests among the students

$n$ students, $n=3$

| A | B | C | |
|---|---|---|---|
| A | B | C | ✗ |
| A | C | B | ✗ |
| B | A | C | ✗ |
| B | C | A | ✓ |
| C | A | B | ✓ |
| C | B | A | ✗ |

$n! = 1 \times 2 \times 3 \times \cdots \times n$ is the # permutation

What is: $1 + 2 + 3 + \cdots + (n-1)$




# possible pairs: $(n-1) + (n-2) + (n-3) + \cdots + 1$

$1 + 2 + 3 + \cdots + (n-1) =$ # possible pairs we can make on n things.

$$1 + 2 + 3 + \cdots + (n-1) = \#\text{ pairs}$$

$$= \binom{n}{2} = C^n_2$$

$$= \text{"n choose 2"}$$

$$= \frac{(n-1)(n-1+1)}{2}$$

$$= \frac{(n-1)n}{2}$$

we already know this

$$1 + 2 + 3 + \cdots + \underbrace{(n-1)}_{m}$$

$$= 1 + 2 + 3 + \cdots + m = \frac{m(m+1)}{2}$$

$$= \frac{(n-1)(n-1+1)}{2}$$

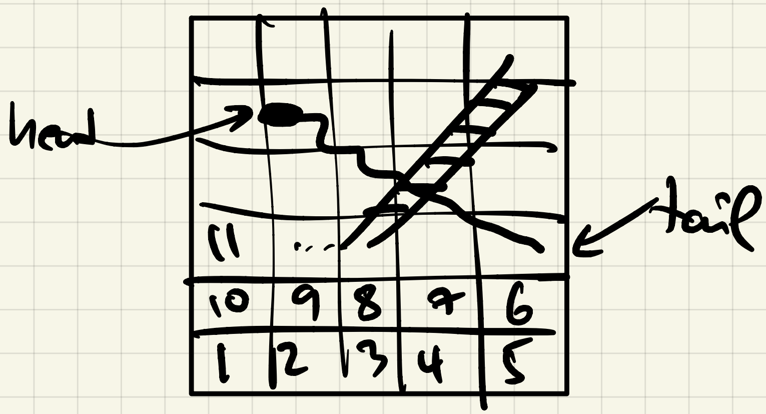
$$= \frac{(n-1)n}{2}$$

Example:

$$1 + 2 + 3 + \cdots + 10 = \frac{10 \times 11}{2}$$

$$1 + 2 + 3 + \cdots + 9 = \frac{9 \times 10}{2}$$

# Snakes & Ladders




head > tail

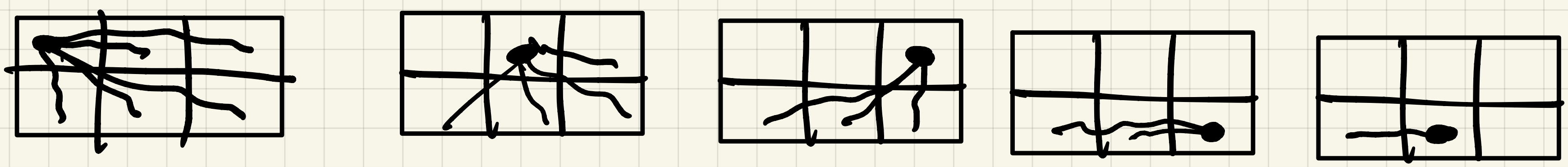
In how many ways can I place 1 snake on a board with n squares?

## Example $n = 6$

Snake is defined by a pair of squares.

| | | |
|---|---|---|
| 6 | 5 | 4 |
| 1 | 2 | 3 |

How many ways can I make a pair of squares?



5    4    3    2    1

General:

$(n-1)$    $(n-2)$    . . . . .    1

Whenever we talk about pairs, we mean $\underline{\underline{\text{unordered}}}$ pairs.

$$\binom{n}{2} = \frac{(n-1)n}{2} = \# \ \underline{\underline{\text{unordered}}} \text{ pairs}$$

# ordered pairs is $2\binom{n}{2} = 2\,\frac{(n-1)n}{2}$

$$= (n-1)n$$

$$1 + 2 + 3 + \cdots + n = \sum_{i=1}^{n} i$$

$$1 \times 2 \times 3 \times \cdots \times n = \prod_{i=1}^{n} i$$

In general

$$\sum_{i=a}^{b} f(i) \qquad\qquad \prod_{i=a}^{b} f(i)$$

Evaluate $f(i)$ for $i = a, a+1, a+2, \ldots, b$, then

add them up

multiply them