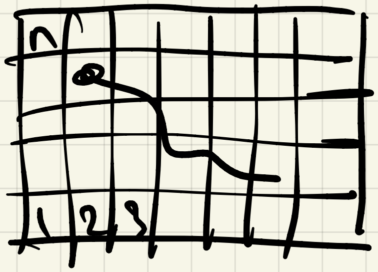
Review:

$n! = 1 \times 2 \times 3 \times \dots \times n$ [# permutations]

$1 + 2 + 3 + \cdots + (n-1) = \frac{(n-1)n}{2} = \binom{n}{2}$ "n choose 2"

[# unordered pairs]

$\binom{n}{2}$ themes



Two squares define a snake

How many snakes are possible

abstraction → How many ways I can choose a pair of square

$$\binom{n}{2}$$

Product rule:

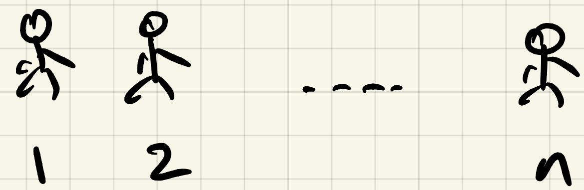
		# ways
1.	Choose a Square ...	n
2.	choose another square ...	$(n-1)$
		$n(n-1)$

overcounting

" ordered pairs "

$$\frac{n(n-1)}{2} = \binom{n}{2}$$

Let's say we have n people

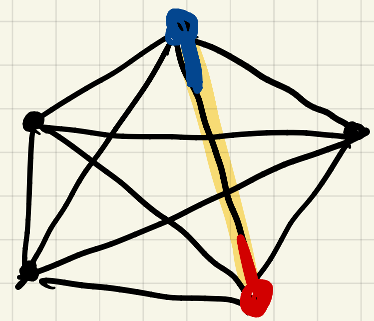


- They all shook hands
- How many handshakes?

a handshake is uniquely defined by an unordered pair. $\binom{n}{2}$

Given a graph with n vertices and all possible edges, how many edges

Example: $n=5$



An edge is defined by 2 "Unordered" vertices

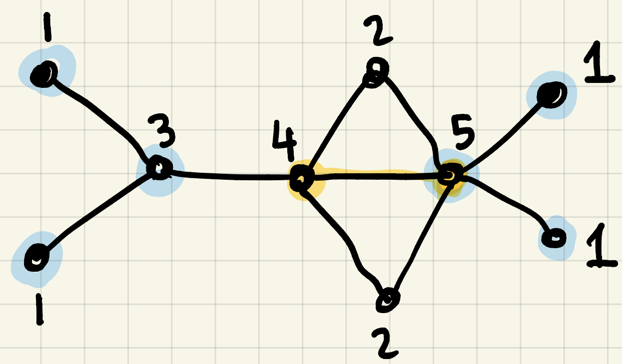
$$\binom{n}{2} = \frac{(n-1)n}{2}$$

In our example above

$$n=5 \Rightarrow \binom{n}{2} = \binom{5}{2} = \frac{5 \times 4}{2} = 10$$

Another Explanation:

		#ways	in general
1.	choose a vertex ----	5	n
2.	choose another vertex ----	4	$(n-1)$
		5×4	$n(n-1)$



- Add up all the degrees: $1+1+3+4+2+2+5+1+1 = 20$
- Every edge is counted exactly twice: once from each end

Let d_i be degree of vertex i (assume n vertices)

$$d_1 + d_2 + \cdots + d_n = \sum_{i=1}^{n} d_i = 2 \times \#\text{edges}$$

Handshake Lemma

$\binom{n}{2}$ in a set setting.

A set is a collection of things "informal"

$S = \{1, 2, 3, \ldots, n\}$ $\quad |S| = n$

How many subsets of size 2 does S have

$\{1,2\}, \{1,3\}, \ldots, \{2,3\}, \ldots, \{n-1, n\}$

$\binom{n}{2}$ = # subsets of size 2

Remark: $\{1,2\} = \{2,1\}$

Example: $n=4$, $S = \{1,2,3,4\}$

$\{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}$

$$\binom{4}{2} = \frac{4 \times 3}{2} = 6$$

Boys & Girls

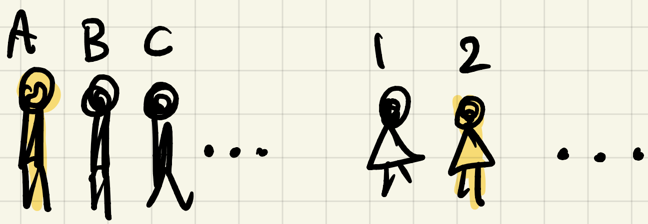
m boys

n girls

In how many can I make a couple? (a boy / girl)

It's not $\binom{m}{2}$ or $\binom{n}{2}$ or $\binom{m+n}{2}$ or $\binom{m}{n}$

couple		# ways
	1. choose a boy ----	m
	2. choose a girl ----	n
		$m \times n$



No overcounting!

(A, 2) shows up exactly once

Intersting observation: $$\underbrace{\binom{m}{2} + \binom{n}{2} + mn}_{\text{Addition rule}} = \binom{m+n}{2}$$

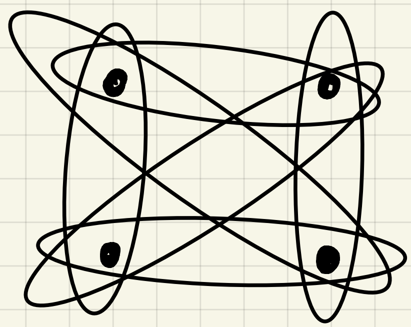
Be careful how we look at pairs :

$$\binom{n}{2} \neq \frac{n}{2}$$

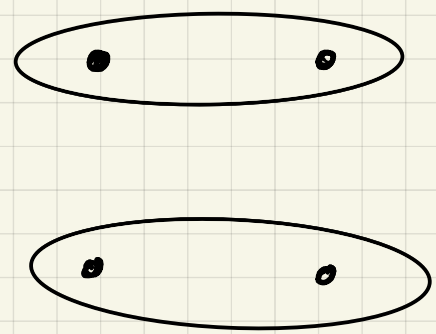
$\binom{n}{2}$: # ways we can select a pairs

$\frac{n}{2}$: (when n even) number of pairs that exist simultaneously

<u>$n=4$</u>

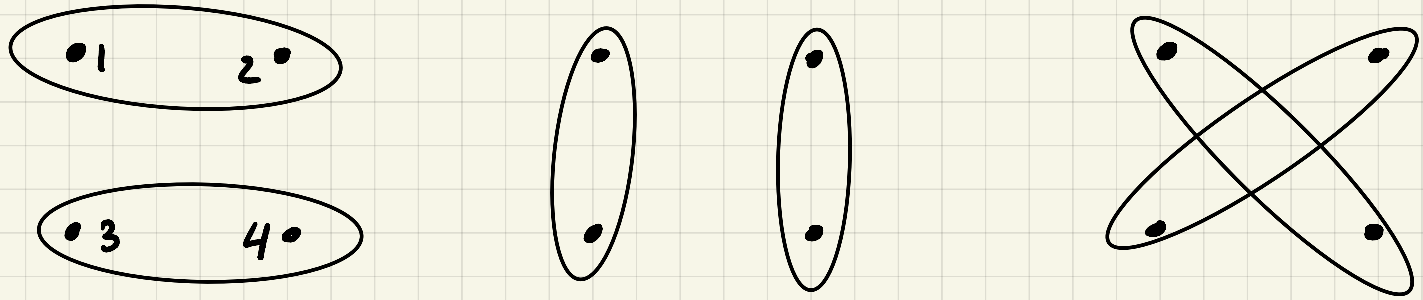


$\binom{4}{2} = 6$



$\frac{4}{2} = 2$

In how many ways can we make simultaneous pairs?



3 ways

In general, if we have $2n$ people, in how many can we make teams of 2.

We don't have an existing abstraction or framework

go to scratch

			# ways
2 team 1	1.	choose a person -----	$2n$
	2.	choose another person -----	$2n-1$
2 team 2	3.	choose " " ----	$2n-2$
	4.	choose " " ----	$2n-3$
⋮	⋮		
2 team n	$2n-1$.	choose " " ----	2
	$2n$.	choose " " ----	1

$n!$

$$\frac{(2n)!}{2^n n!}$$

I am over counting? ~~Yes~~ Yes

try $n=2$

$n=2 \Rightarrow \dfrac{(2\times 2)!}{2^2 2!} = \dfrac{4!}{4\times 2} = \dfrac{24}{8} = 3$