What is Discrete Math about?

First, it's not "Discreet" as in secretive.
In fact, every one should know Discrete Math
because it's the math for computer science
and EVERY DAY

Some general topics studied in Discrete Math:

- Counting / combinatorics (why we count)
  - Discover patterns / structure
  - Understand complexity of object we are dealing with
  - Fun

- Proofs (No math without proofs)
  - Need to establish facts
  - For CS, prove correctness / properties of algorithms
- Sets / relations / functions (general tools)
- Number theory (study of integers & their properties)

  e.g. Cryptography is heavily based on number theory

- Graphs (not plots)
  - very important tool in CS to model pairwise relations
    e.g. networks (communication, social, roads, ...)
  - Graph algorithms such as finding shortest path between two locations heavily rely on graph theory

Example questions / settings in Discrete Math and how they relate to above general topics.

- Birthday Paradox: Given a few people, there is a high probability that two share a birthday
  - Such fact can be established by counting.
  - Essential for idea of "collision" in hashing for instance

- Number Games: Collatz & Ducci
  - Collatz: Start with a positive integer x

$$x \text{ even} : x \longleftarrow x/2$$
$$x \text{ odd} : x \longleftarrow 3x+1$$

Repeat

Examples of the Collatz game:

10, 5, 16, 8, 4, 2, 1

17, 52, 26, 13, 40, 20, 10, ..., 1

Conjecture: Regardless where we start, we will always reach 1

[No proof yet!] Paul Erdös: "Mathematics is not yet ready for such problems"!

But it looks so easy that a 5-year old would understand it!
Yet, it's too deep!

- **Ducci**: Start with a sequence of n integers $(a_1, a_2, \dots, a_n)$ and update it as follows:

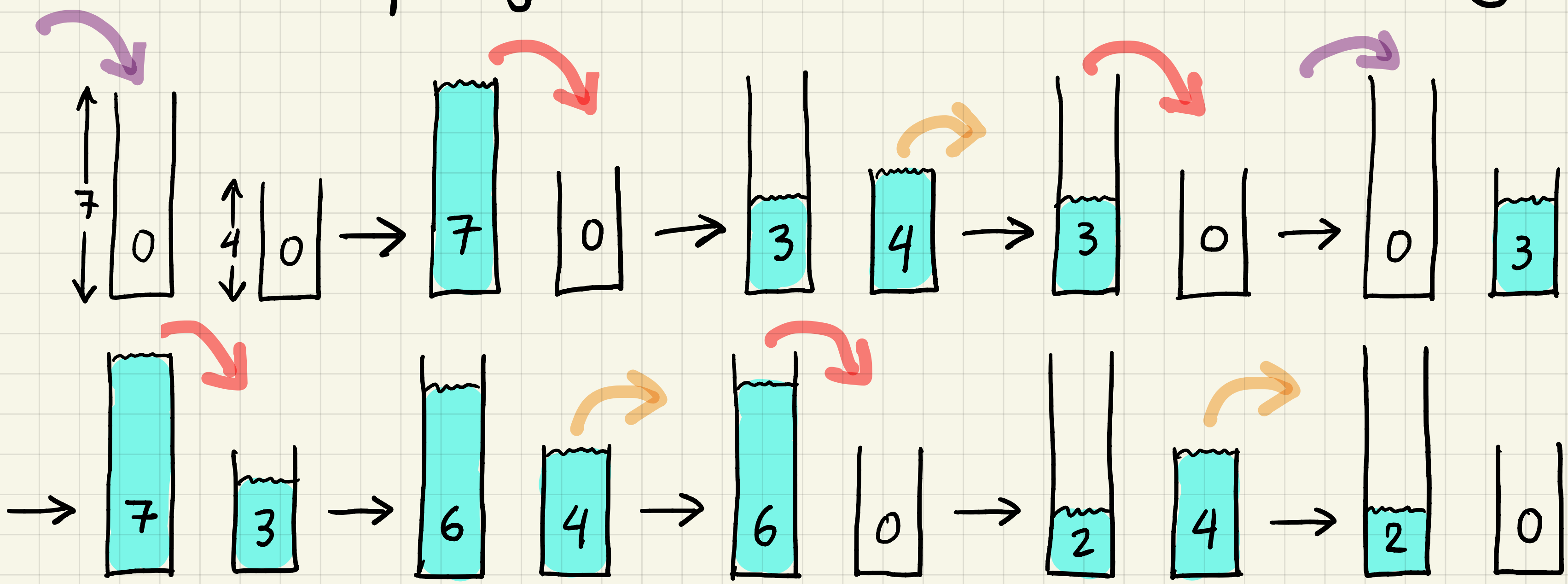
$$(|a_1 - a_2|, |a_2 - a_3|, \dots, |a_n - a_1|)$$

Example: $n=4$

$$(1,2,3,4) \to (1,1,1,3) \to (0,0,2,2)$$
$$\to (0,2,0,2) \to (2,2,2,2) \to (0,0,0,0)$$

Fact: When $n=4$, we will always reach $(0,0,0,0)$

- **Water Juggling**: Given two containers, one with capacity 7 and another with capacity 4, can we measure exactly 2?



This is related to the fact that 7 and 4 are co-prime, they share no prime factors

$$7x - 4y = 2 \quad (x = 2,\ y = 3)$$

- **Sequences**:

  The most famous sequence is perhaps the Fibonacci sequence:

  0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

  "Each number is the sum of the previous two"

  If F_n is the n^{th} Fibonacci number, then

$$\underbrace{F_n = F_{n-1} + F_{n-2}}_{\text{This is called "Recurrence"}} \qquad n \geq 2$$

because it defines the sequence recursively in terms of itself.

We often use **proofs by Induction** to establish properties of sequences ← later

- **Sums**: Consider $T_n = 1 + 2 + 3 + \dots + n$
(add all pos. integers $\leq n$)

Continuous version
$f(x) = x$
Find $\int_0^n f(x)\,dx$

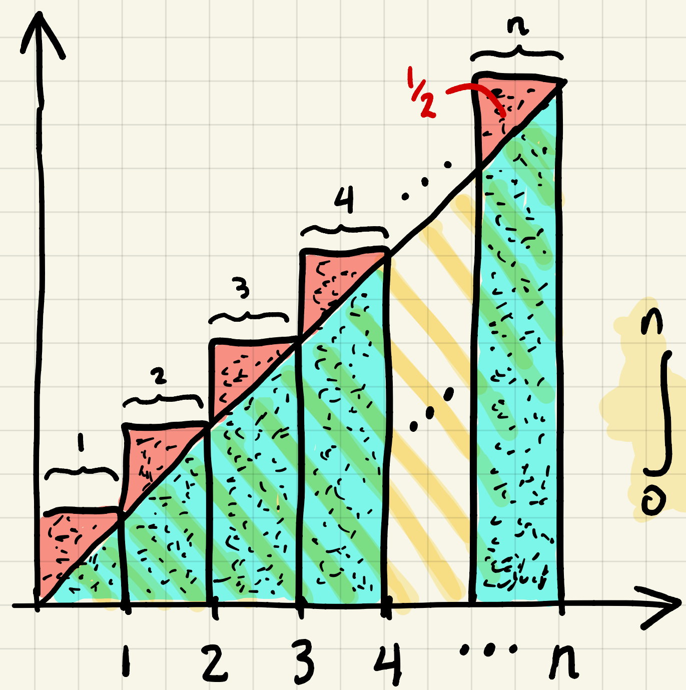
Examples: $n = 1: T_1 = 1$
$n = 2: T_2 = 3$
$n = 3: T_3 = 6$
$n = 4: T_4 = 10$

Recurrence? $T_n = T_{n-1} + n$

Area under curve

Discrete setting:

$$\underbrace{1 + 2 + 3 + \dots + n}_{\text{area of rectangles}} = \frac{n^2}{2} + \underbrace{\text{total red Area}}_{n \times 1/2}$$

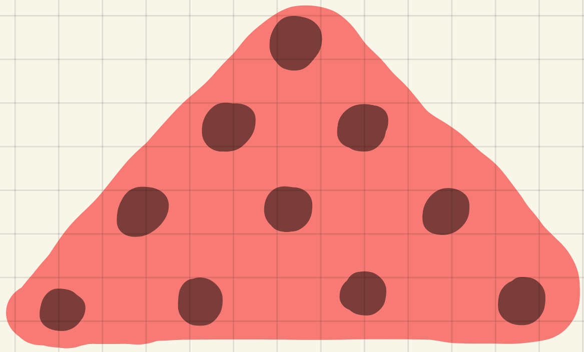


$$\int_0^n x\,dx = \frac{n^2}{2}$$

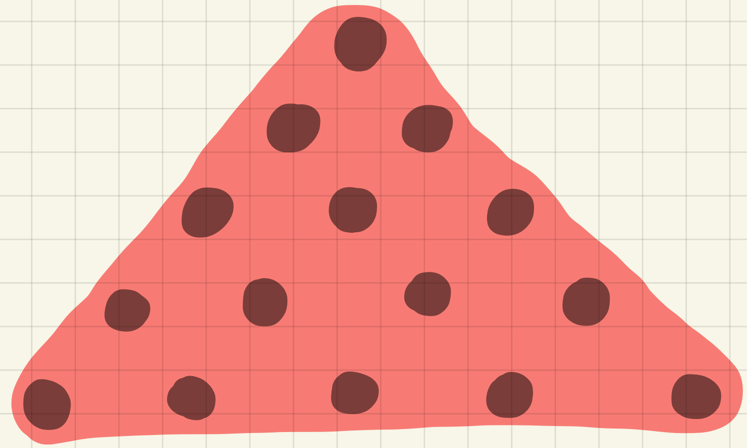
$$T_n = 1 + 2 + 3 + \dots + n = \frac{n^2}{2} + \frac{n}{2} = \frac{n(n+1)}{2}$$

This sum is related to counting pairs (later)

$T_n =$ n^{th} triangular number



$T_4 = 10$



$T_5 = 15$

Triangular numbers in real life ?

e.g. phase out medication, take 6 pills on the first day, 5 pills on the next, etc...

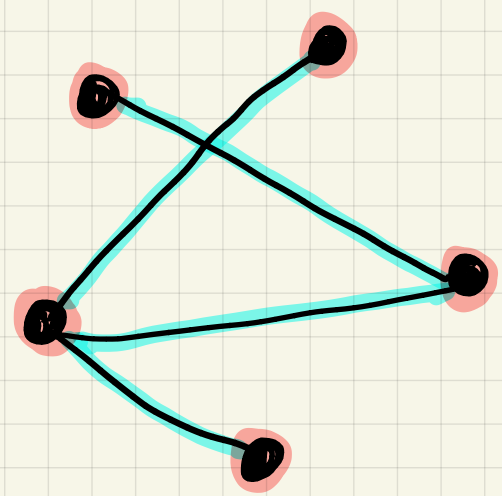
$$T_6 = 6 + 5 + 4 + 3 + 2 + 1 = 21 \text{ pills !}$$

- **Graphs**: Pairwise relation

  vertices : represented visually by dots

  edges : represented visually by arcs connecting vertices

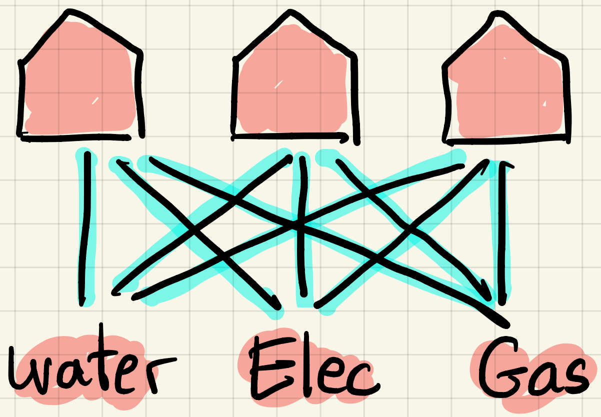
Example:



5 vertices

4 edges

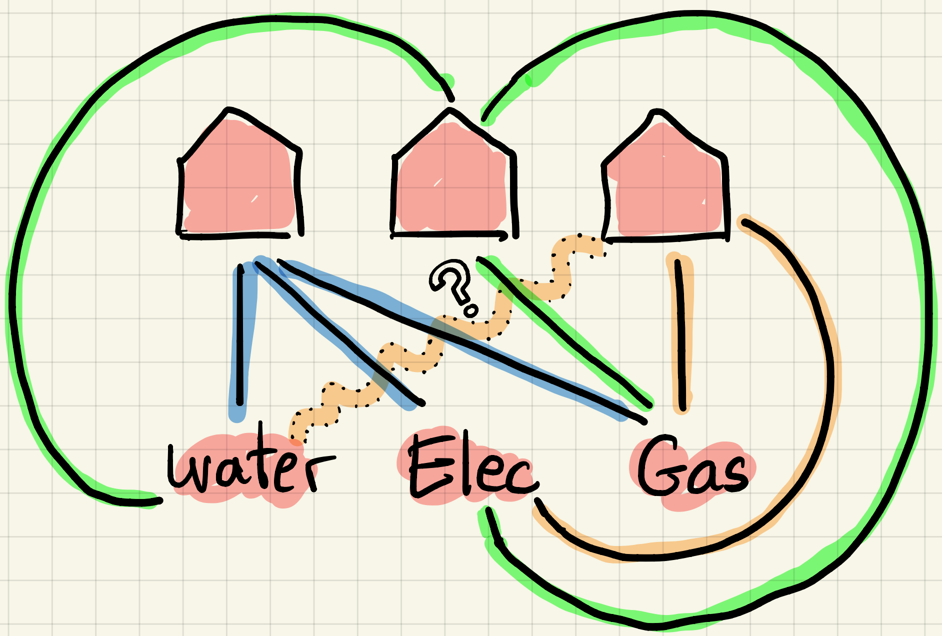
Nasty Neighbors: Connect 3 houses to 3 utilities



6 vertices

9 edges

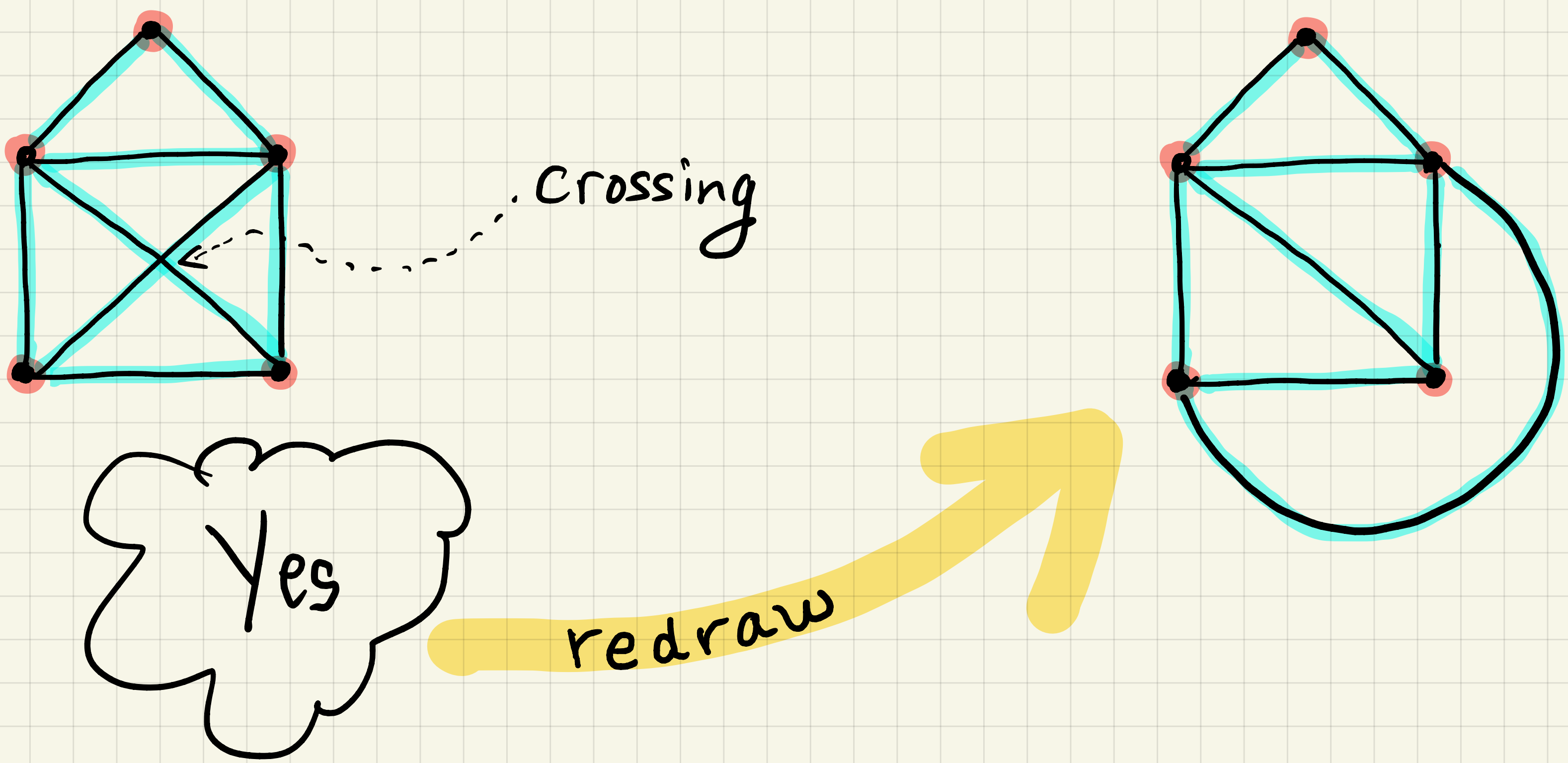
But can we do it without crossings?



It can't be done: We say graph is NOT planar

Graph is planar means we can draw it in the plane without edges crossing.

Is this planar ?



Next: <u>Count</u> vertices & edges to establish patterns!!