Example 3: Prove $\sqrt{2}$ is irrational.

by contradiction

$P: \sqrt{2} \notin \mathbb{Q}$

$\neg P: \sqrt{2} \in \mathbb{Q}$

$\sqrt{2} \in \mathbb{Q} \Rightarrow \sqrt{2} = \frac{a}{b}$ (a & b are integers and $\frac{a}{b}$ is irreducible)

$\Rightarrow 2 = \frac{a^2}{b^2} \Rightarrow a^2 = 2b^2 \Rightarrow a^2$ is even $\Rightarrow$ a is even.

$\sqrt{2} \in \mathbb{Q} \Rightarrow 2 = \frac{a^2}{b^2} \Rightarrow b^2 = \frac{a^2}{2} = \frac{2k \times 2k}{2} = \frac{4k^2}{2} = 2k^2$

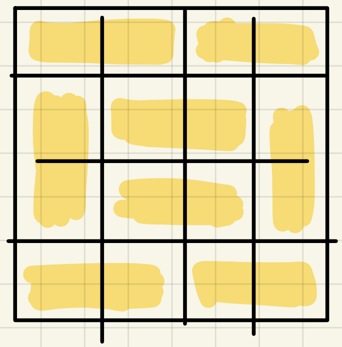
$\Rightarrow b^2$ is even $\Rightarrow$ b is even

a is even and b is even $\Rightarrow \frac{a}{b}$ is reducible

$\sqrt{2} \in \mathbb{Q} \Rightarrow \underbrace{\left(\frac{a}{b} \text{ is reducible and } \frac{a}{b} \text{ is irreducible}\right)}_{\text{False}}$

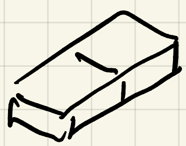
More proofs

Consider a $n \times n$ board. (n is even)

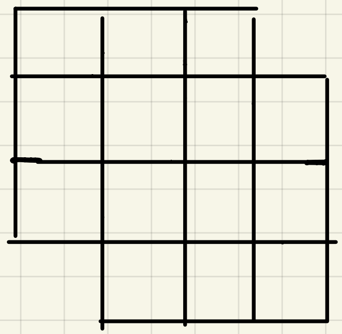


(Example $n=4$)

Dominos:



Can we cover the board with Dominos?



If we delete 2 opposite corners can we still cover the board with Dominos ?

P: Two opposite corners are deleted

Q: Board is <u>not</u> coverable

$P \Rightarrow Q$

Prove $(P \Rightarrow Q)$ is true.

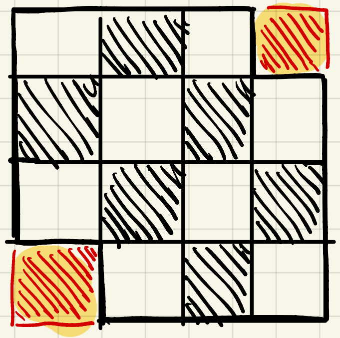
<u>Proof by contradiction:</u>

How would I start:

Start with $P \wedge \neg Q$ (that's the negation of $P \Rightarrow Q$)

<u>Parity argument</u>: make every square even or odd
2 adjacent squares have different parity.

e.g. Square is even if sum of its coordinates is even



n even means that

even square = # odd squares

P : Two opposite corners deleted

$\Rightarrow$ two square of the same parity deleted

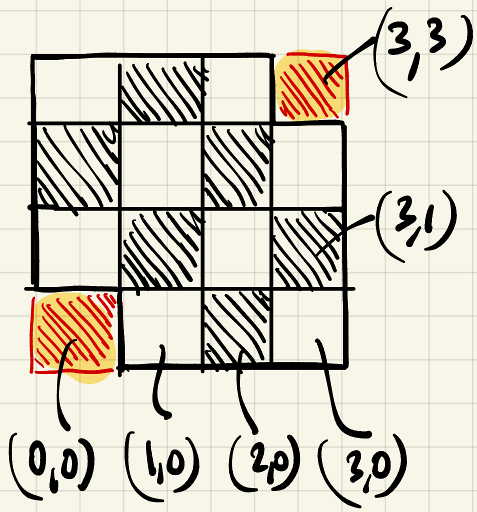
$\Rightarrow$ # even square $\neq$ # odd squares

$\neg Q$: board is Coverable

$\Rightarrow$ every Domino covers two adjacent square with different parity

$\Rightarrow$ # even square $=$ # odd square

$P \wedge \neg Q \Rightarrow$ False . Therefore $(P \Rightarrow Q)$ is true


(3,3)
(3,1)
(0,0)
(1,0)
(2,0)
(3,0)

Prove by contradiction that primes are infinite.

Prime are finite $\Rightarrow$ the set of prime numbers is finite

$\Rightarrow$ say it's $\mathbb{P} = \{ p_1, p_2, p_3, \ldots, p_k \}$ where $k \in \mathbb{N}$

Magic (Light bulb): $n = 1 + \prod_{i=1}^{k} p_i$, $n \in \mathbb{N}$

$n \notin \mathbb{P}$ (not prime)

$\Rightarrow$ n is not divisible by any of the prime numbers (division has remainder 1)

a contradiction since every integer can be factored into primes.