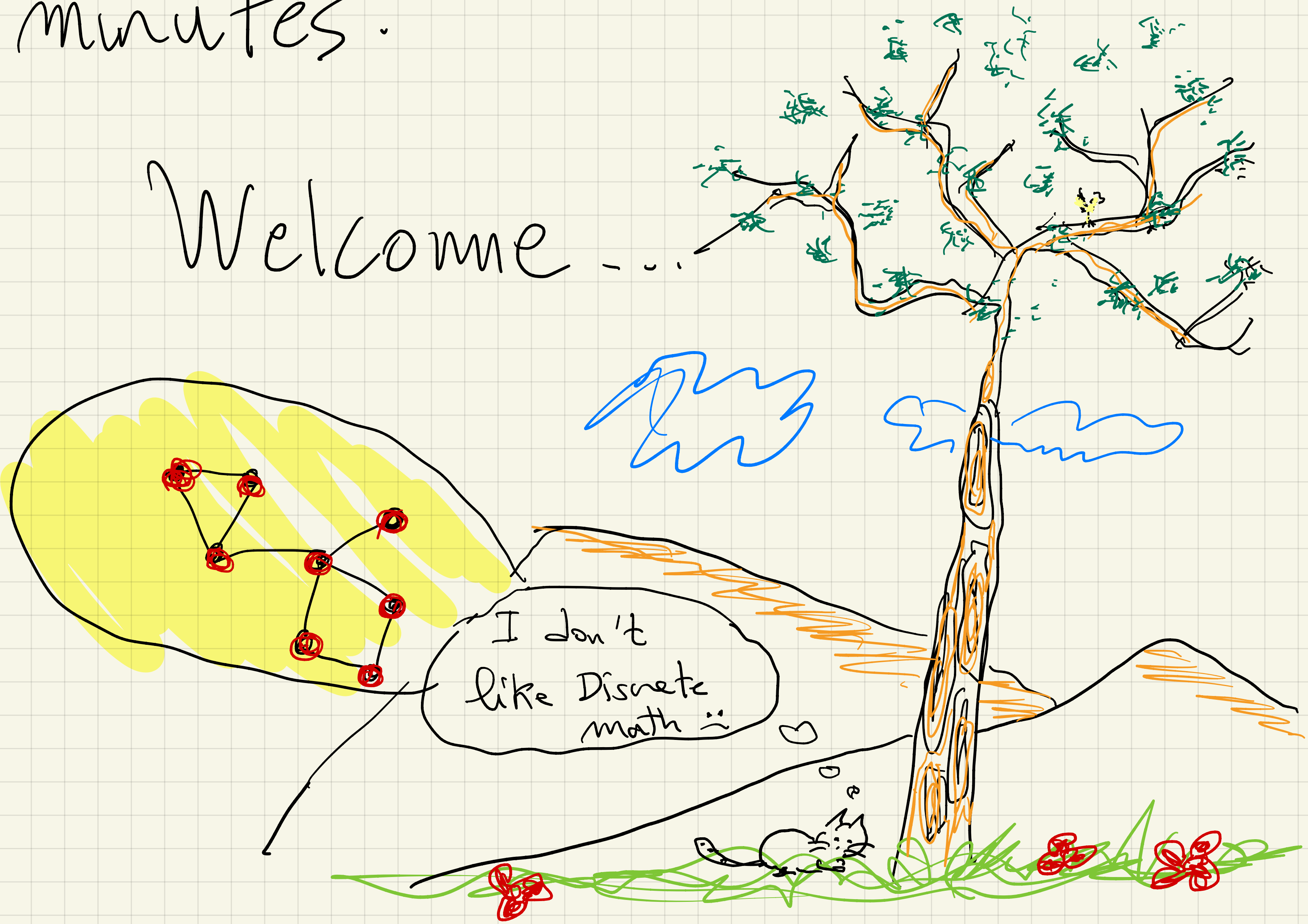
We will be starting in few minutes.

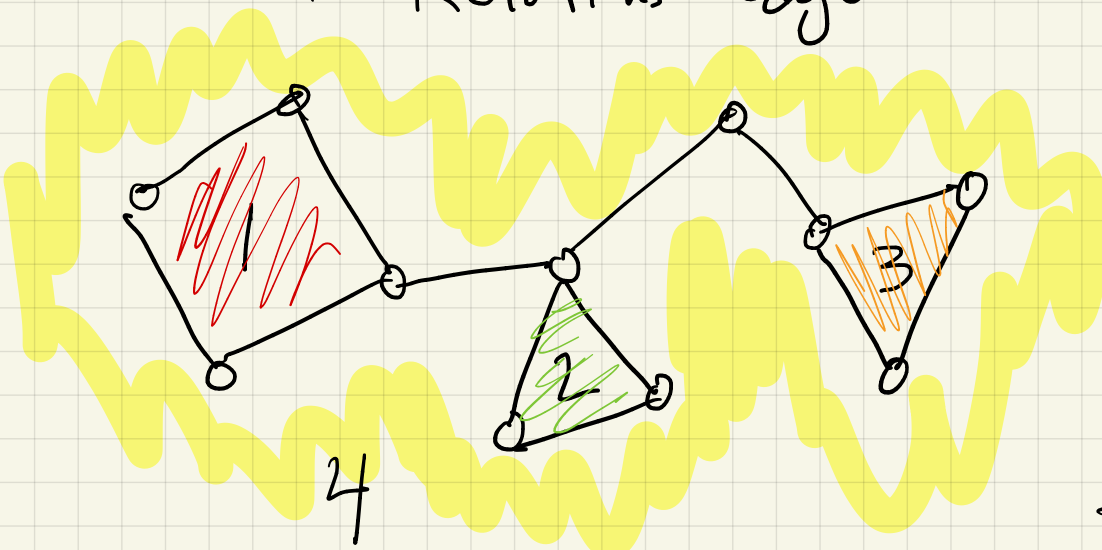
Welcome ...



Counting:

Graph: Abstraction to represent pairwise relation

- Entities: represented by vertices
- Relations: edges



Planar: Edges do not overlap

face: (planar graphs) "area" you can move around without crossing any edges (there is always the outer face)

$f = 4$

$v = 11$

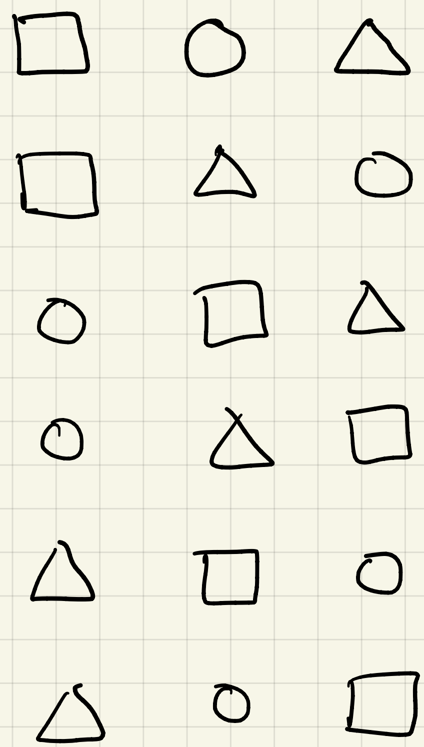
$e = 13$

$$v - e + f = 2$$

1) Counting helps establish structure. (Euler's formula)

2) Counting helps determine complexity of objects we are dealing with.

Example:



6 permutations

$n = 3$

$n! = 1 \times 2 \times 3 = 6$

$\#$ permutations $= n!$ where $n = \#$ objects

permutation : $n! = 1 \times 2 \times 3 \times \cdots \times n$

$n = 3$: $1 \times 2 \times 3 = 6$

$n = 4$: $1 \times 2 \times 3 \times 4 = 24$

$n = 5$: $1 \times 2 \times 3 \times 4 \times 5 = 120$

$\vdots$

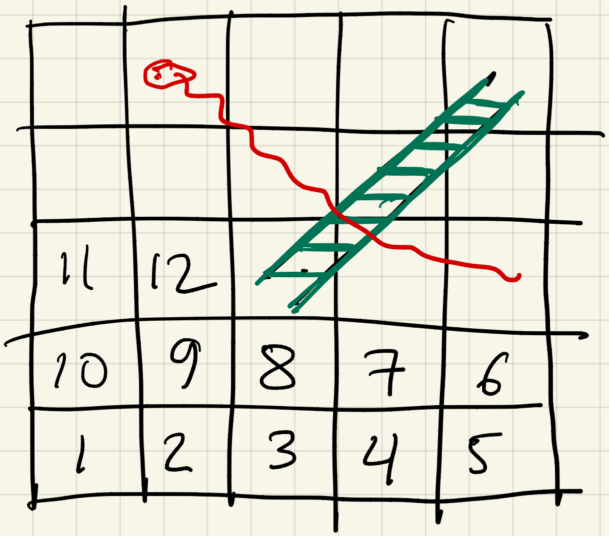
$n = 10$: $1 \times 2 \times 3 \times \cdots \times 10 = 3628800$

$n = 100$: 158 digit number (large)

Can't list all permutation to look for certain properties, we have to do it in a smarter way.

Example Counting: Snakes & Ladders

head > tail

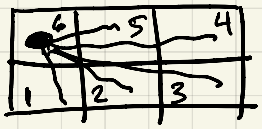


Pros: No thinking required

Cons: Placement of snakes & ladders so that it's not boring.

Question: In how many ways can I place one snake on a board with n squares?

$n=6$



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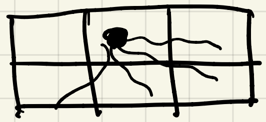
Snakes with head on square 6



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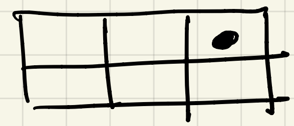
Snakes with head on square 5



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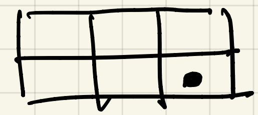

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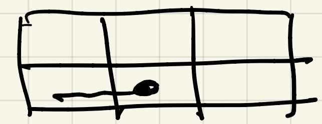
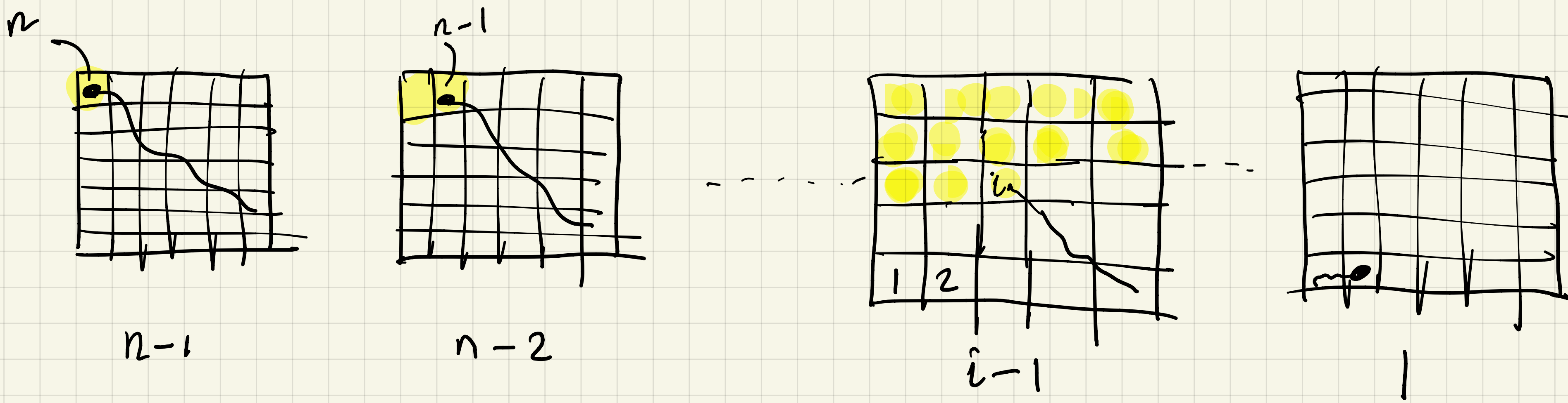


1

We have considered all placements of the head. For each scenario, we counted all possible snakes.

\# of possible placements of 1 snake: $5+4+3+2+1=15$

Generalization for n



$\#$ possible ways: $(n-1) + (n-2) + \cdots + 1$

Ex: $n = 6$: $5 + 4 + \cdots + 1 = 15$

$$1 + 2 + 3 + \cdots + (n-1)$$

$$=$$

$$(n-1) + (n-2) + (n-3) + \cdots + 1$$

Summations & Products notations

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$1 + 2 + 3 + \cdots + \underbrace{(n-1)}_{m} = \frac{m(m+1)}{2} = \frac{(n-1)n}{2}$$

$$1 + 2 + 3 + \cdots + (n-1) = \frac{(n-1)n}{2} = \binom{n}{2}$$

"n choose 2"

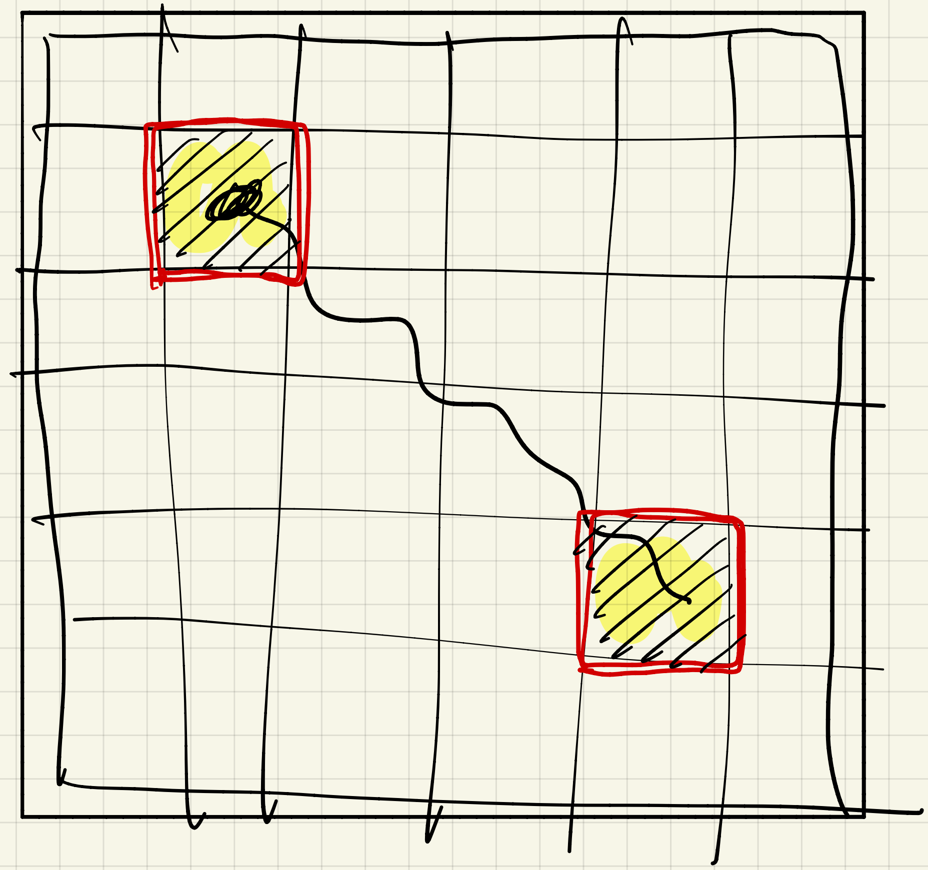
$$\binom{n}{2} = C^{n}_{2}$$

Why is

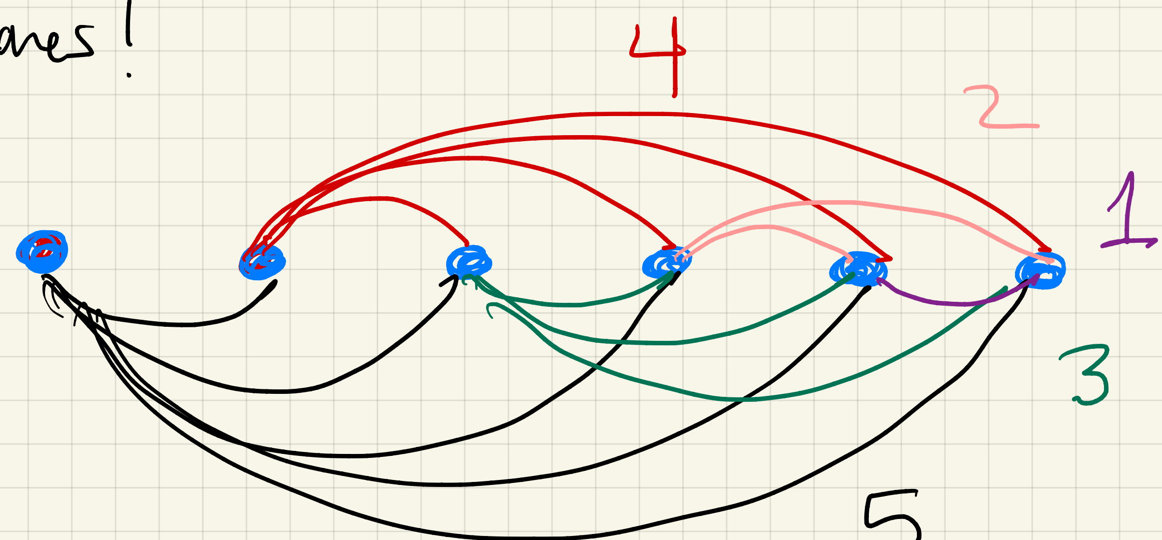
$$\frac{(n-1)\,n}{2}$$

called

"n choose 2"?



Essentially we are choosing 2 squares out of n squares!



$n=6$

$$1 + 2 + 3 + \cdots + (n-1) = \sum_{i=1}^{n-1} i$$

(notation)

$$\sum_{i=a}^{b} E$$

b ← upper bound

a ← lower bound

"Replace i in E by all values from a to b and add them up"

Our example: $\sum_{i=1}^{n-1} i$ $\quad \begin{cases} a = 1 \\ b = n-1 \\ E = i \end{cases}$

$$\left.\begin{array}{l} i=1 : E = i = 1 \\ i=2 : E = i = 2 \\ \vdots \\ i=n-1 : E = i = n-1 \end{array}\right\} +$$

"unfolding the sum" $\quad 1 + 2 + \cdots + (n-1)$

Why is the sum notation Σ good?

because it eliminates ambiguity!!

example: Add the first 10 terms of the following sequence

$$\underbrace{1 + 2}_{+1} + \underbrace{ + 4}_{+2} + \underbrace{ +}_{+3 \, ?} \text{---------}$$

one possibility: $7 + 11 + \cdots$

another possibility: $8 + 16 + 32 + \cdots$

this is what I meant!

$$\sum_{i=0}^{9} 2^i = \underbrace{1 + 2 + 4 + \cdots\cdots}_{10 \text{ terms}}$$

i	$E = 2^i$
0	1
1	2
2	4
3	8
$\vdots$	$\vdots$
9	512

$+$ $\quad 1+2+4+8+\cdots+512$

$$\#\text{ terms} = \text{upper bound} - \text{lower bound} + 1$$

$$= 9 - 0 + 1 = 10 \checkmark$$

$$n! = 1 \times 2 \times 3 \times \cdots \times n = \prod_{i=1}^{n} i$$

Remember:

Empty Sum $= 0$

Empty product $= 1$

Special Case $(n=0)$

$$\frac{n(n+1)}{2} = 1 + 2 + \cdots + n = \sum_{i=1}^{n} i$$

$$n! = 1 \times 2 \times \cdots \times n = \prod_{i=1}^{n} i$$

empty sum: $\sum_{i=1}^{0} i$ (upper bound < lower bound)

empty prod: $\prod_{i=1}^{0} i$ (" " ")

Empty product: $a^0 = 1$

$a^1 = a$

$a^2 = a \times a$

$a^3 = a \times a \times a$

Empty sums & emty products

Sum

```
s ← ?
for i ← 1 to n
    s ← s + i
return s
```

Product

```
p ← ?
for i ← 1 to n
    p ← p × i
return p
```

$$\sum_{i=1}^{n} i \qquad\qquad n! = \prod_{i=1}^{n} i$$

for each program

- Replace ? by appropriate number
- Determine the return value when $n = 0$

Take home message

$$n! = \prod_{i=1}^{n} i = \#\text{ permutations on } n \text{ objects}$$

$$1 + 2 + \cdots + (n-1) = \frac{n(n-1)}{2} = \sum_{i=1}^{n-1} i =$$

$\#$ pairs given n objects

Empty sum (e.g. $n = 1$ in above sum) is 0

Empty product (e.g. $n = 0$ in above prod.) is 1

so $0! = 1$