CSCI 150 Discrete Mathematics
Homework 0
Due 09/04/09

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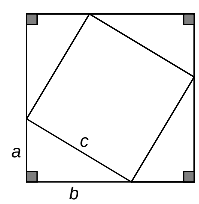
Problem 1: Pythagoras Theorem
We argued in class that there is no mathematics without proofs, and presented a proof for the famous Pythagoras theorem that most of you have been using for years: $c^2 = a^2 + b^2$.



We have the following areas:

- Outer square: $(a+b)^2$
- Inner square (why is it a square?): c^2
- Each triangle: $ab/2$

Therefore,

$$(a+b)^2 = c^2 + 4ab/2$$

which gives $c^2 = a^2 + b^2$.

(a) Find either on your own, or by searching the Internet, another proof of the Pythagoras theorem.

(b) A Pythagorean triple (a, b, c) where a, b, and c are positive integers is such that $a^2 + b^2 = c^2$. Given two positive integers m and n with $m > n$, show that $a = 2mn$, $b = m^2 - n^2$, and $c = m^2 + n^2$ form a Pythagorean triple.

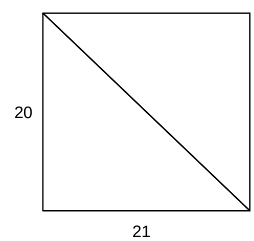
(c) A primitive Pythagorean triple (a, b, c) is a Pythagorean triple that cannot be simplified by dividing a, b, and c by the same integer. For instance, $(3, 4, 5)$ is primitive, but $(6, 8, 10)$ is not. Fix n to be 1 in part (b) above. What seems to be a property of m to generate a primitive Pythagorean triple?

Problem 2: False proof

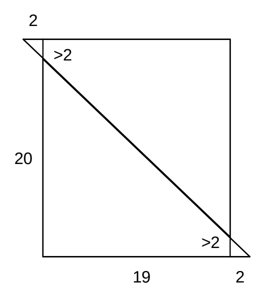
A proof should establish the truth of a proposition with absolute certainty. In practice, however, many proofs contain errors: overlooked cases, logical slips, algebraic mistakes, etc... But in a well-written proof, even if there is a bug, one should at least be able to point out a specific statement that does not logically follow.

False Theorem. $420 > 422$

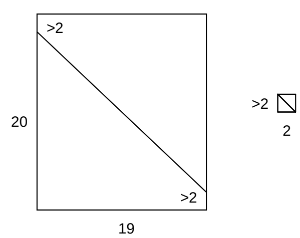
proof: We will demonstrate this fact geometrically. We begin with a 20x21 rectangle, which has area 420.



Now we cut along the diagonal as indicated above and slide the upper piece parallel to the cut until it has moved exactly 2 units leftward. This leaves a couple of stray corners which are 2 units wide and just over 2 units high.



Finally, we snip off the two corners and place them together to form an additional small rectangle:



Now we have two rectangles, a large one with area just over (20+2)x19=418 and a small one with area just over 2x2=4. Thus, the total area of the resulting figure is a bit over 418+4=422. By conservation of area, 420 is equal to just a little bit more than 422. □

Where is the error?