

# CSCI 150 Discrete Mathematics

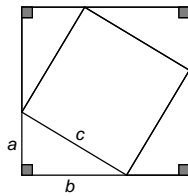
## Homework 0

### Solution

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#### Problem 1: Pythagoras Theorem

We argued in class that there is no mathematics without proofs, and presented a proof for the famous Pythagoras theorem that most of you have been using for years:  $c^2 = a^2 + b^2$ .



We have the following areas:

- Outer square:  $(a + b)^2$
- Inner square (why is it a square?):  $c^2$
- Each triangle:  $ab/2$

Therefore,

$$(a + b)^2 = c^2 + 4ab/2$$

which gives  $c^2 = a^2 + b^2$ .

(a) Find either on your own, or by searching the Internet, another proof of the Pythagoras theorem.

**Solution:** check the following link, <http://www.cut-the-knot.org/pythagoras/>

(b) A Pythagorean triple  $(a, b, c)$  where  $a$ ,  $b$ , and  $c$  are positive integers is such that  $a^2 + b^2 = c^2$ . Given two positive integers  $m$  and  $n$  with  $m > n$ , show that  $a = 2mn$ ,  $b = m^2 - n^2$ , and  $c = m^2 + n^2$  form a Pythagorean triple.

**Solution:**  $a^2 + b^2 = (2mn)^2 + (m^2 - n^2)^2 = 4m^2n^2 + m^4 + n^4 - 2m^2n^2 = m^4 + n^4 + 2m^2n^2 = (m^2 + n^2)^2 = c^2$

(c) A primitive Pythagorean triple  $(a, b, c)$  is a Pythagorean triple that cannot be simplified by dividing  $a$ ,  $b$ , and  $c$  by the same integer. For instance,  $(3, 4, 5)$

is primitive, but  $(6, 8, 10)$  is not. Fix  $n$  to be 1 in part (b) above. What seems to be a property of  $m$  to generate a primitive Pythagorean triple?

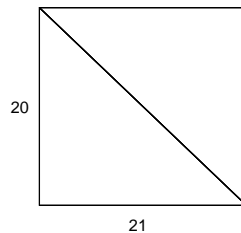
**Solution:** If  $m = 1$ , then a triple has the following form  $(2m, m^2 - 1, m^2 + 1)$ . If  $m$  is odd, all numbers in the triple are even and, therefore, the triple is not primitive because we can divide all the numbers in the triple by 2. If  $m$  is even, then we can observe that  $m^2 - 1$  and  $m^2 + 1$  are two consecutive odd numbers. Two consecutive odd numbers cannot have a common factor and, therefore, we cannot simplify the triple. We conclude that when  $m$  is even, the triple is primitive.

**Problem 2: False proof**

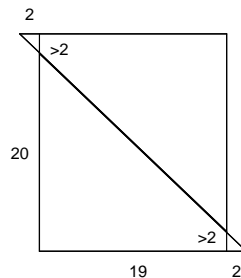
A proof should establish the truth of a proposition with absolute certainty. In practice, however, many proofs contain errors: overlooked cases, logical slips, algebraic mistakes, etc... But in a well-written proof, even if there is a bug, one should at least be able to point out a specific statement that does not logically follow.

**False Theorem.**  $420 > 422$

*proof:* We will demonstrate this fact geometrically. We begin with a  $20 \times 21$  rectangle, which has area 420.

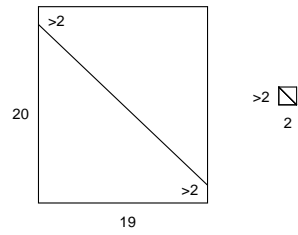


Now we cut along the diagonal as indicated above and slide the upper piece parallel to the cut until it has moved exactly 2 units leftward. This leaves a couple of stray corners which are 2 units wide and just over 2 units high.



Finally, we snip off the two corners and place them together to form an additional small rectangle:

Now we have two rectangles, a large one with area just over  $(20+2) \times 19 = 418$  and a small one with area just over  $2 \times 2 = 4$ . Thus, the total area of the resulting figure is a bit over  $418 + 4 = 422$ . By conservation of area, 420 is equal to just a little bit more than 422.  $\square$



Where is the error?

**Solution:** The error is the following statement: “This leaves a couple of stray corners which are 2 units wide and just **over** 2 units high.” The correct statement should be: “This leaves a couple of stray corners which are 2 units wide and just **below** 2 units high.”