CSCI 150 Discrete Mathematics
Homework 1

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Due Mon 2/7/2024 (before midnight)

Exercises

1. Express the sum $1+3+5+7+\ldots+127$ using the notation $\sum_{i=a}^{b} f(i)$. First, figure out the number of terms in the sum, then use this information to determine the lower and upper bounds. For instance, a could be 1 and b could be the number of terms, or a could be 0 and be b could be the number of terms minus 1. Finally, find the expression inside the sum that would result in the desired terms. Once you figure out your sum notation, find the answer using the technique of splitting the sum.

2. Let k be an integer. When can we claim the following?

$$k^2 = \sum_{i=1}^{k} k$$

   Experiment with that expression for several values of k before answering the question.

   Note: You might try encoding this as a for loop in Python.

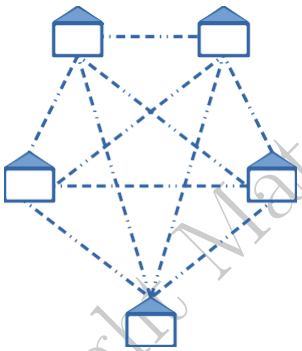
3. Assume x and y are fixed parameters. Express the following using a $\prod$ notation:

$$\text{Pock}(n) = (1-x)(1-xy)(1-xy^2)\ldots(1-xy^{n-1})$$

What is Pock(0) and why?

4. How many five-digit numbers can we make if we can use each of the digits 1, 3, 5, 7, and 9 exactly once? Optional: What if we change the digits to 0, 2, 4, 6, and 8?

5. Given the following map, in how many ways can you visit all the houses (each house must be visited exactly once, and you can start with anyone you want)? (why are the answers to this question and the previous question the same?)



Extra challenge (you don't have to submit this): Repeat the above if the road between two of the houses is closed (it does not matter which two).

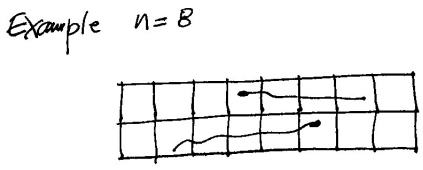
6. The Zelder family has a newborn. The parents want to choose a name and a middle name in such a way that the initials will make three unique letters that appear in alphabetical order (call this a monogram):

$$- - Z$$

For instance ABZ is allowed, but AAZ (not unique) and BAZ (not alphabetical) are not. In how many ways can the parents choose the monogram? (*Hint*: Recall the example of placing one snake on a board where we divided the snakes into disjoint categories based on the head of the snake. Do the same here, and divide the monograms into disjoint categories based on which letter they start with).

7. Repeat the above if letters of the monogram are not required to be in alphabetical order.

Problem
Consider a regular $2 \times n$ snakes and ladders board. We are interested in placing two snakes, but each must be in a separate row (the rule that head must be greater than tail is still required). An example is shown below:



(a) Use the product rule to figure out the number of ways can we place the two snakes. Explain your reasoning. The answer must be for a general n.

(b) The board has n columns. If a head or a tail of any snake falls in a given column, we say that the column is *involved*. In the example above, four columns are involved. Give an example of placing the two snakes where two columns are involved, and another one where three columns are involved.

(c) Find the number of ways you can place the two snakes if

- two columns are involved
- three columns are involved
- four columns are involved

Hint: There are multiple ways of doing this, but mostly the product rule should be helpful.

(d) What does the addition rule guarantee about your answer for (a) and the three answers in (c)? Verify.