CSCI 150 Discrete Mathematics
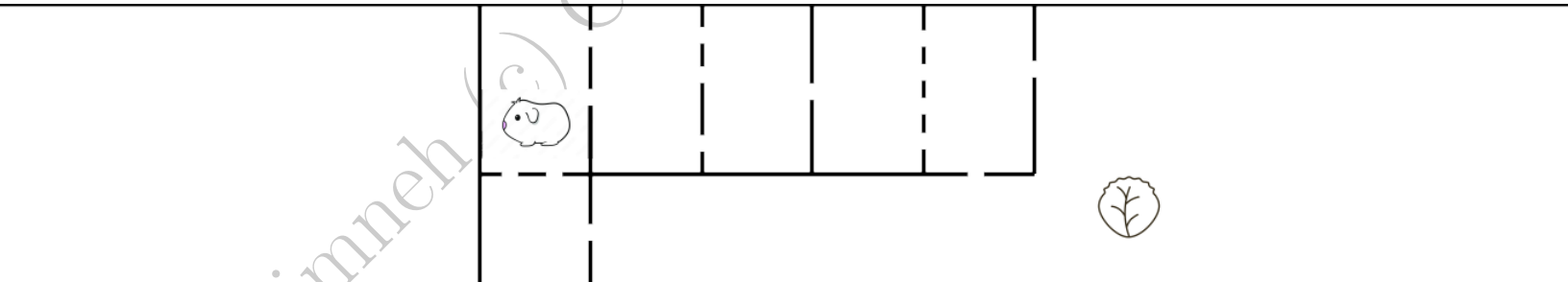# Homework 2

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Due Wed. 2/14/2024 (before midnight)

Exercises

1. Find in how many ways my guinea pig Chanel can get to the lettuce. Which two important principles have you used to find the answer? (if you have not used some important principles, then rethink the problem)

2. How many patterns can you make with 2 digits, followed by a letter, followed by a digit, if your pattern cannot start with 0?

3. We place 561 points on the circumference of a circle. How many cords of the circle do these points make?

4. Consider a lottery ticket with 60 numbers. To buy a ticket, you need to choose 5 of these numbers. Each lottery ticket costs \$1. How big must the prize be to justify buying all possible tickets?

5. In the lottery above, how many possible tickets are there if you must choose a sixth number for the "power ball"? Find the answer using the product rule in two ways:

   - choose the power ball first, then your five numbers
   - choose your five numbers first, then the power ball

   Verify that both ways yield the same answer and state an equality that generalizes the result by changing 60 to n and 5 to k. Verify this equality algebraically.

6. Consider the set $S = \{1, 2, 3, \ldots, n\}$. How many subsets of S contain 1? How many subsets of S contain 1 and have size k?

7. How many words of length 7 can you make using the alphabet $\{a, b, c \ldots, z]$ (words don't have to be in the dictionary).

8. Consider the number of ways we can seat n people on k chairs. Is this number greater than or less than the number of ways we can seat k people on n chairs? Justify your answer.

Problem
(a) Assume you have n objects numbered $1, 2, \ldots, n$. Call a good permutation one that places object 1 before object n (otherwise, call the permutation bad). How many good permutations are there? You must count the good permutations by following the approach outlined below (and the product rule):

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                                                            # ways
-----------------------------------------------------------------
1. choose two positions out of n positions ................

2. place objects 1 and n in these 2 positions .............

3. permute the other objects using remaining positions .....
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Simplify your answer as much as possible after applying the product rule (you should obtain something surprisingly simple and interesting, hopefully...)

(b) Use what you have discovered in part (a) to solve the following: Imagine you have a pair of gloves, a pair of socks, and a pair of boots. These are labeled GL, GR, SL, SR, BL, BR, where the first letter indicates glove, sock, or boot, and the second letter indicates left or right. In how many ways can you put these on?

Hint: We are looking at permutations of 6 objects here, but SL must be put on before BL, and SR must be put on before BR.