

CSCI 150 Discrete Mathematics
Homework 3
Due 10/09/09

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Problem 1: Neighbors

(a) Consider 20 persons sitting on one side of a rectangular table. How many ways can we choose 3 persons, no two of whom are neighbors?

Hint: the choice of three persons partitions the table into 4 sets of people S_1 , S_2 , S_3 , and S_4 , where $|S_1| + |S_2| + |S_3| + |S_4| = 17$ and $S_2 \neq \emptyset$ and $S_3 \neq \emptyset$.

(b) What if the 20 persons are sitting on a round table?

Hint: thinking about a partitioning of the table into three sets S_1 , S_2 , and S_3 , such that $|S_1| + |S_2| + |S_3| = 17$ and non of the sets is empty does not work. Why?

Another Hint: select a person to be the head of the table (this defeats the purpose of a round table, but it serves the math). If this person is not chosen, then we have to choose three persons on a rectangular table with 19 persons. If on the other hand that person is chosen, then we have to choose two persons on a rectangular table with 17 persons.

Problem 2: A party

Consider a party of 60 persons. Assume that each one knows at least m others.

(a) How large must m be to guarantee that 3 persons know each other?

(b) How large must m be to guarantee that 4 persons know each other?

Hint: Use the inclusion exclusion principle, as illustrated in class.

Problem 3: Beware of Pokemon

There is a contest with 40 Pokemons. There are 18 Pokemons who like to fight in the sky, and 23 who like to fight on ground. Several of them like to fight in water. The number of those who like to fight in the sky and on ground is 9. There are 7 Pokemons who like to fight in the sky and in water, and 12 who like to fight on ground and in water. There are 4 Pokemons who like to fight

in the sky, on ground, and in water. How many Pokemons like to fight in water?

Problem 4: A great idea for Pigeonhole

We select 38 even positive integers, all less than 1000. Prove that there will be two of them whose difference is at most 26.