

CSCI 150 Discrete Mathematics  
Homework 4  
Due 10/27/09

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**Problem 1: The binary system**

Show by induction that every natural number can be represented by a sum of **distinct** powers of 2.

(a) regular induction: *Hint*: when you assume the property is true for  $n - 1$ , come up with an expression for  $n - 1$ . You will find that it is a sum of powers of 2 with different coefficients (of course you don't know what the coefficients are). So

$$n - 1 = \sum_{i=0}^k a_i 2^i$$

Consider the first coefficient that is zero. Use this information and the fact that  $\sum_{i=0}^{j-1} 2^i = 2^j - 1$ , which we proved in class, to make the inductive step work.

(b) strong induction. *Hint*: let  $k$  be the largest integer such that  $n = m + 2^k$ . Use strong induction on  $m$  to obtain the result.

**Problem 2: What is that sum?**

(a)

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{(n-1) \cdot n}$$

(b)

$$0 \cdot \binom{n}{0} + 1 \cdot \binom{n}{1} + 2 \cdot \binom{n}{2} + \dots + (n-1) \cdot \binom{n}{n-1} + n \cdot \binom{n}{n}$$

Experiment, conjecture the value, and then prove it by induction. The second one is hard, so ask me.

**Problem 3: Some Fibonacci identities**

Show the following:

(a)  $F_2 + F_4 + F_6 + \dots + F_{2n} = F_{2n+1} - 1$   
(hint: use induction)

(b)  $F_{n+1}^2 - F_n^2 = F_{n-1}F_{n+2}$   
(hint: don't use induction)

(c)  $F_{n-1}F_{n+1} - F_n^2 = (-1)^n$   
(hint: use  $F_{a+b+1} = F_{a+1}F_{b+1} + F_aF_b$  on  $a + b + 1 = 2n - 1$  for different values of  $a$  and  $b$ )

**Problem 4: Recurrence**

Find an explicit value for

$$a_n = 3 \cdot a_{n-1} - 2 \cdot a_{n-2}$$

given that  $a_0 = 0$  and  $a_1 = 1$

**Problem 5: Paving a garden path**

A path is 2 meters wide and  $n$  meters long. It is to be paved using paving stones of size  $1\text{m} \times 2\text{m}$ . In how many ways can the paving be accomplished?

*Hint:* Rabbits may cross the path.