Exercises (Due Thursday 3/21/2019)

1. Show that $A - B^C = A \cap B$, recall that $B^C$ is $B$’s complement and
   
   $$A - B = \{ x : x \in A \land x \notin B \}$$

2. Consider the set of all cups in the entire world. In this set countable or uncountable?

3. What about the set of all cups that will ever be made?

4. You are given a function $f : \mathbb{N} \to S$ that is onto. Is $S$ countable or uncountable?

5. Let $S$ be a countably infinite set. Consider the set $T = S \cup \{ x \}$ where $x \notin S$. Is $T$ countable? Provide a proof for your answer.

6. Consider the set of all sets of powers of 2. Is this set countable or uncountable?

7. What about the set of all finite sets of powers of 2?

8. A classroom has 40 people. Show that at least 4 of them must be born on the same month.

9. Consider the set $S = \{1, 2, \ldots, 100\}$. Choose six numbers from $S$ at random. Show that among those six numbers, there are two numbers $x$ and $y$, such that $|x - y| \leq 19$.

10. The numbers from 1 to 10 are written down in some random order. Show that there are three consecutive numbers that add up to at least 14. [Show 15 for extra credit].
**Problems** Due Monday 3/25/2019

**Problem 1**

(a) Consider the set $S = \mathbb{R} - \mathbb{Q}$. Show that this set is uncountable, possibly using proof by contradiction. *Hint*: $(A - B) \cup B = A \cup B$.

(b) Use the diagonal method to show that the set of all functions $f : \mathbb{Q} \to \{0, 1\}$ is uncountable.

**Problem 2**

(a) Verify both distributive laws of $\land$ and $\lor$:

$$P \land (Q \lor R) = (P \land Q) \lor (P \land R) \quad \text{and} \quad P \lor (Q \land R) = (P \lor Q) \land (P \lor R)$$

using a truth table.

(b) Use the distributive law to show that $(A - B) \cup (B - A) \subset A \cup B$.

**Problem 3**

The city of New York designed a new ONE WAY sign, shown below (measurements in feet).

(a) A car drove by the sign and splashed it with seven spots of mud (see an example at www.cs.hunter.cuny.edu/~saad/courses/dm/hw/spots.png). Show that two spots must be within a distance of 1.5 feet on the sign. *Hint*: You can choose to see the back of the sign by going to www.cs.hunter.cuny.edu/~saad/courses/dm/hw/NYChint.png

(b) [City wide challenge] A good citizen was passing by and saw what happened. He cleaned one spot as it was covering the text on the sign. Show that among the six remaining spots, two of them are still within a distance of 1.5 feet.

(c) If another spot gets cleaned, can we still maintain the same claim about the remaining five spots?