CSCI 150 Discrete Mathematics
Homework 5

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Due Wed. 3/13/2024 (before midnight)

1. Think of the most fascinating theorem that you have ever encountered. It could be a simple one, or it could be very sophisticated. Whatever the statement of that theorem is, let's call it P. Consider the following proposition:

$$\text{pigs can fly} \Rightarrow P$$

   - Why is the above proposition true?
   - How come this is not a proof of P itself?

2. For each of the following, find whether it is always true, always false, or neither:

$$0 \wedge P$$
$$1 \wedge P$$
$$0 \vee P$$
$$1 \vee P$$
$$0 \Rightarrow P$$
$$1 \Rightarrow P$$
$$P \vee \neg P$$
$$P \wedge \neg P$$
$$P \Rightarrow \neg P$$

3. Let P and Q be two propositions and consider the following proposition:

$$(P \Rightarrow Q) \vee (Q \Rightarrow P)$$

Show that the above proposition is always true:

- by means of a truth table:

P Q	$P \Rightarrow Q$	$Q \Rightarrow P$	$(P \Rightarrow Q) \vee (Q \Rightarrow P)$
0 0			
0 1			
1 0			
1 1			

- by replacing any proposition of the form $X \Rightarrow Y$ by $\neg X \vee Y$ and using the associative property of $\vee$.

4. An integer n is a multiple of 3 if $n = 3k$, and is not a multiple of 3 if $n = 3k + 1$ or $n = 3k + 2$ (these are all possible cases), where $k \in \mathbb{Z}$. Consider the following statement:

$$\forall n \in \mathbb{Z},\ (n^2 \text{ is not a multiple of } 3 \Rightarrow n \text{ is not a multiple of } 3)$$

(a) Prove the statement is true by working with its contrapositive.

(b) By finding n^2 for each possible case listed above for n, conclude a stronger result:

$$\forall n \in \mathbb{Z},\ (n \text{ is a multiple of } 3 \Leftrightarrow n^2 \text{ is a multiple of } 3)$$

5. Prove by a counter example that the following statement is false

$$\forall n \in \mathbb{Z},\ (n^2 \text{ is a multiple of } 4 \Rightarrow n \text{ is a multiple of } 4)$$

6. Prove by contradiction that $\sqrt{3} \notin \mathbb{Q}$.

7. Consider $(P \wedge Q) \Rightarrow R$ and $P \Rightarrow (R \vee \neg Q)$. Show that they are equivalent:

- by means of a truth table
- by reasoning about when exactly each of them is false (without a truth table).

8. Assume that all of the following are true

$$(P \vee Q) \Rightarrow R$$

$$Q \vee R$$

$$R \Rightarrow P$$

Show by contradiction that P is true.