Problem 1: Sliding window algorithm
The sliding window algorithm with unbounded sequence numbers is given in the notes. Upon receiving an \( RN \), the sender checks if \( RN > SN \) and slides the window. Similarly, upon receiving an \( SN \), the receiver checks if \( SN = RN \) and slides the window. Describe the modifications needed (if any) for these two rules when the sequence numbers are sent modulo \( p \) (bounded).

**ANSWER:** The rule for the receiver does not need to change. However, the rule for the sender will not work as it is possible to receive a smaller subsequent number after performing the modulo \( p \) operation. More precisely, the sender needs to check if the received \( RN \) is equal to any of \( SN + 1 \mod p \ldots SN + n \mod p \), and slide the window accordingly.

Problem 2: Modulus without FIFO
Assume that a sliding window algorithm is used but the FIFO property does not hold. Instead, consider only frames that are sent and received (possibly with error). If a frame is the \( i^{th} \) frame sent and the \( j^{th} \) frame received, then \( |i - j| \leq K \). Note that when \( K = 0 \), we have the FIFO property. Here’s an example scenario with \( K = 3 \).

(a) Show that if the receiver receives \( SN \) at time \( t \), then

\[
RN(t) - n - K \leq SN \leq RN(t) + n - 1
\]

**ANSWER:** If frame \( SN \) was received at time \( t \), consider the time \( t_0 < t \) at which the received frame \( SN \) was transmitted. We can establish three inequalities similar to those derived in class (refer to note 5). The first two inequalities remain the same:

\[
SN(t_0) \leq SN \leq SN(t_0) + n - 1 \text{ (window)}
\]

\[
SN(t_0) \leq SN(t) \leq RN(t) \text{ (non-decreasing)}
\]
The third inequality was:

\[ RN(t) \leq SN(t_0) + n \] (FIFO)

which is not true anymore. This inequality says that frame \( SN(t_0) + n \) cannot have been sent before \( t_0 \) and, therefore, by the FIFO property, cannot have been received before \( t \). Thus, \( RN(t) \leq SN(t_0) + n \). With the FIFO property now replaced with the more general property, this is not true anymore. In fact, up to \( K \) frames transmitted after frame \( SN \) can now be received before time \( t \). This requires a proof.

Let's prove it by contradiction. Assume that more than \( K \) frames are transmitted after frame \( f \), but received before frame \( f \). For the property to be satisfied, at least one frame transmitted before frame \( f \) must be received after frame \( f \). Let this frame be \( f' \). Looking at \( f' \), we find that more than \( K \) frames transmitted after \( f' \) are received before \( f' \). Therefore, applying the argument recursively, we can establish the existence of a frame \( f'' \). Eventually, we reach a point where we cannot carry the argument further, and hence a contradiction.

Therefore, by time \( t \), the receiver will see at most \( K \) consecutive frames starting at frame \( SN(t_0) + n \). Therefore, \( RN(t) \) is at most \( SN(t_0) + n + K \). Putting the three inequalities together, we obtained the desired result.

(b) Show that if the sender receives \( RN \) at time \( t \), then

\[ SN(t) - Kn \leq RN \leq SN(t) + n \]

**ANSWER:** Again, we establish inequalities similar to those derived in class (refer to note 5). The first one is trivial and remains the same:

\[ RN \leq SN(t) + n \] (window)

The second one was:

\[ SN(t) \leq RN \] (FIFO)

which is not true anymore. This inequality says that by time \( t \), the sender cannot have received a request number greater than \( RN \), because of the FIFO property and because the receiver sends \( RN \) in increasing order. As argued above, with the FIFO property replaced by the more general property, up to \( K \) subsequent requests from the receiver can be received at the sender before \( RN \). Depending on when the receiver decides to send a request (see part (c) for instance), each subsequent \( RN \) can advance up to \( n \) from the previous \( RN \) (it cannot advance more than \( n \) because the sender cannot slide the window before receiving a new request). Therefore, the sender can receive request \( RN + Kn \) by time \( t \). Thus, \( SN(t) \leq RN + Kn \). Putting the two inequalities together, we obtain the desired result.

(c) Assume that the receiver slides the window by one position at a time and sends the appropriate \( RN \), but nothing is lost so it either arrives error-free or with errors. Show that part (b) can be modified as follows:

\[ SN(t) - K \leq RN \leq SN(t) + n \]
**ANSWER:** Using the argument in part (b), the subsequent requests are contiguous, and hence the sender will receive at most request $RN + K$.

(d) Based on parts (a) and (c), what would be a sufficient modulus to use for this non-FIFO setting?

**ANSWER:** The receiver needs to distinguish $2n + K$ values, but as described in class, it really needs to distinguish only $n + m + K$ values. The sender needs to distinguish $n + 1 + K \leq n + m + K$. Therefore, a modulus of $n + m + K$ is enough.

**Problem 3: Link initialization protocols**

(a) Consider an unbalanced Master-Slave protocol for link initialization in which either $A$ or $B$ can be the Master at any point in time. In other words, both $A$ and $B$ can initialize and disconnect the link. Show by constructing a sequence of INIT and DISC messages (and appropriate ACKs) that, in the presence of message loss, $A$ can receive a DISC before determining that the link is up.

**ANSWER:**

(b) Consider the same protocol above but with the following modification: the last ACK (whether ACKI or ACKD) is piggybacked on every INIT or DISC. Show that this modification avoids the situation in part (a) assuming that ACKs are acted upon first.

**ANSWER:**

(c) Construct a sequence of INIT and DISC messages (with appropriate ACKs and piggybacked ACKs) in such a way that, in the presence of message loss, $B$ considers the link to have gone through an up period followed by a down period, while $A$ constantly thinks that the link is down.
(d) Show that the balanced Master-Slave protocol described in class avoids the situation in part (c) assuming that ACKs are acted upon first.

**ANSWER:** see notes.