# CSCI 415 Data Communication Networks Homework 7 in progress...

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## Problem 1: Application of Little's theorem

Two communication nodes 1 and 2 send packets to another node 3. The average lengths for packets generated at 1 and 2 are  $L_1$  and  $L_2$  bytes respectively. The links connecting node 1 and node 2 to node 3 have bandwidths  $\mu_1$  and  $\mu_2$  bps (bits per second) respectively. Node 3 can processes one packet at a time (no queue) and spends an average of  $P_i$  seconds on packets from node *i*. Assume a steady state and a rate of  $\lambda_1$  and  $\lambda_2$  packets/sec for nodes 1 and 2, respectively.

(a) What is the average number  $\rho_1$  of packets of node 1 that are being processed by node 3 at any given time?

(b) What is the average number  $\rho_2$  of packets of node 2 that are being processed by node 3 at any given time?

(c) What should be the relation between  $\rho_1$  and  $\rho_2$ ?

(d) What is the average delay per packet in node 3?

(e) What is the average number  $N_1$  of packets of node 1 that are in the system at any given time?

(f) What is the average number  $N_2$  of packets of node 2 that are in the system at any given time?

(g) What is the average delay per packet in the system?

## Problem 2: A Little paradox

Consider Example 1 in note 9. Applying Little's theorem we have the following:

$$N = \lambda T$$

$$N_Q = \lambda W$$

$$\rho = \lambda \frac{L}{\mu}$$

$$N = N_Q + \rho$$

where:

- N is the average number of packets in the node
- $N_Q$  is the average number of packets in the queue
- $\rho = N N_Q$  is the average number of packets being transmitted

Multiplying by L, the average size of the packet, we have:

$$NL = \lambda LT$$
$$N_Q L = \lambda LW$$
$$\rho L = \lambda L \frac{L}{\mu}$$
$$NL = N_Q L + \rho L$$

This means that NL is the average number of bits in the node,  $N_QL$  is the average number of bits in the queue, and  $\rho L$  is the average number of bits being transmitted. However, applying Little's theorem to the bits in the transmitter, we have:

- $\lambda L$  is the rate in bits
- $1/\mu$  is the average delay per bit

Hence,  $\lambda L/\mu = \rho$  is the average number of bits being transmitted. One of these statements must be wrong, which one and why?

## Problem 3: Packet switching and circuit switching

A communication line is capable of transmitting at a rate of 50 Kpbs will be used to accommodate 10 sessions each generating a Poisson traffic at a rate of 150 packets/min. Packet lengths are exponentially distributed with mean 1000 bits.

(a) Explain how the information about the packet lengths is useful in modeling the line as an M/M/1 system.

(b) Calculate the average number of packets in the system, the average number of packets waiting, and the average delay per packet, if packet switching is used.

(c) Repeat (b) if circuit switching is used. What do you conclude?

# Problem 4: Max-Min fairness

Consider six nodes arranged in a ring and connected with unit capacity bidirectional links. There are two sessions from nodes 1, 2, 3, 4, and 5, to node 6, one in each direction. Similarly, there are two sessions from nodes 2, 3, and 4 to node 5, and two sessions from node 3 to node 4. Find the max-min fair rates for these sessions.

*Hint*: There are 9 sessions in each direction. Find how many sessions each link carries in each direction to determine which link must saturate first.