CSCI 415 Data Communication Networks
Homework 7
Solution

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Problem 1: Application of Little's theorem
Two communication nodes 1 and 2 send packets to another node 3. The average lengths for packets generated at 1 and 2 are L_1 and L_2 bytes respectively. The links connecting node 1 and node 2 to node 3 have bandwidths μ_1 and μ_2 bps (bits per second) respectively. Node 3 can processes one packet at a time (no queue) and spends an average of P_i seconds on packets from node i. Assume a steady state and a rate of λ_1 and λ_2 packets/sec for nodes 1 and 2, respectively.

(a) What is the average number ρ_1 of packets of node 1 that are being processed by node 3 at any given time?

ANSWER: The arrival rate of packets from node 1 to node 3 is λ_1 and the average delay per packet in node 3 is P_1. Therefore, the average number of packets of node 1 that are being processes by node 3 is $\lambda_1 P_1$ by Little's theorem.

(b) What is the average number ρ_2 of packets of node 2 that are being processed by node 3 at any given time?

ANSWER: Using the same argument above, we get $\lambda_2 P_2$.

(c) What should be the relation between ρ_1 and ρ_2?

ANSWER: Since node 3 can process at most one packet at a time, $\lambda_1 P_1 + \lambda_2 P_2 \leq 1$.

(d) What is the average delay per packet in node 3?

ANSWER: By Little's theorem, it is $\frac{\rho_1+\rho_2}{\lambda_1+\lambda_2}$.

(e) What is the average number N_1 of packets of node 1 that are in the system at any given time?

ANSWER: The arrival rate is λ_1. The averate delay (in sec) is $8L_1/\mu_1 + P_1$. Therefore, $N_1 = \lambda_1(8L_1/\mu_1 + P_1)$ by Little's theorem.

(f) What is the average number N_2 of packets of node 2 that are in the system at any given time?

ANSWER: Using the same argument above, we get $N_2 = \lambda_2(8L_2/\mu_2 + P_2)$.

(g) What is the average delay per packet in the system?

ANSWER: By Little's theorem, $\frac{N_1+N_2}{\lambda_1+\lambda_2}$.

Problem 2: A Little paradox

Consider Example 1 in note 9. Applying Little's theorem we have the following:

$$N = \lambda T$$
$$N_Q = \lambda W$$
$$\rho = \lambda \frac{L}{\mu}$$
$$N = N_Q + \rho$$

where:

- N is the average number of packets in the node
- N_Q is the average number of packets in the queue
- $\rho = N - N_Q$ is the average number of packets being transmitted

Multiplying by L, the average size of the packet, we have:

$$NL = \lambda LT$$
$$N_Q L = \lambda LW$$
$$\rho L = \lambda L \frac{L}{\mu}$$
$$NL = N_Q L + \rho L$$

This means that NL is the average number of bits in the node, $N_Q L$ is the average number of bits in the queue, and ρL is the average number of bits being transmitted. However, applying Little's theorem to the bits in the transmitter, we have:

- λL is the rate in bits
- $1/\mu$ is the average delay per bit

Hence, $\lambda L/\mu = \rho$ is the average number of bits being transmitted. One of these statements must be wrong, which one and why?

ANSWER: The problem is in the statement that the average number of packets multiplied by the average packet size is equal to the average number of bits. While this is true in the entire system, it is not true for the queue. In particular,

$$N_Q L = \lambda LW$$

but $N_Q L$ cannot be interpreted as the average number of bits in the queue. The reason is that the service model for packets and bits are different. When serving packets, the packet being served in entirely assumed to be taken out of the queue. However, when serving bits, only one bit at a time is taken out of the queue. So, at any point in time, the number of packets in the queue and the number of bits in the queue do not correspond to a multiplicative factor L.

The correct interpretation is the following: NL is the average number of bits in the system (that's correct). ρ is the average number of bits being served. Therefore, the average number of bits in the queue must be $NL - \rho$. The average number of packets in the queue is $N - \rho$. Multiplying this by L we see that we have an additional $(L-1)\rho$ bits on average.

Problem 3: Packet switching and circuit switching
A communication line is capable of transmitting at a rate of 50 Kpbs will be used to accommodate 10 sessions each generating a Poisson traffic at a rate of 150 packets/min. Packet lengths are exponentially distributed with mean 1000 bits.

(a) Explain how the information about the packet lengths is useful in modeling the line as an M/M/1 system.

ANSWER: The length L of a packet is exponentially distributed. Since the transmission time of a packet is L/C where C is the link capacity (a constant), then L/C is exponentially distributed with mean $\overline{L}/C$, that is $\frac{1}{\mu} = 1000/50000 = 0.02$ sec. Now we can model the link as an M/M/1 system because the service times and the interarrival times are exponentially distributed.

(b) Calculate the average number of packets in the system, the average number of packets waiting, and the average delay per packet, if packet switching is used.

ANSWER: $\lambda = 2.5 * 10 = 25$, $\mu = 50$ (packets/sec).

- $N = \frac{\lambda}{\mu-\lambda} = \frac{25}{50-25} = 1$
- $N_Q = N - \rho = 1 - 0.5 = 0.5$
- $T = \frac{1}{\mu-\lambda} = \frac{1}{50-25} = 1/25 = 0.04$ sec

(c) Repeat (b) if circuit switching is used. What do you conclude?

ANSWER: This time $\lambda = 2.5$, and $\mu = 5$ (each session takes 1/10 of the link capacity). The following is per session:

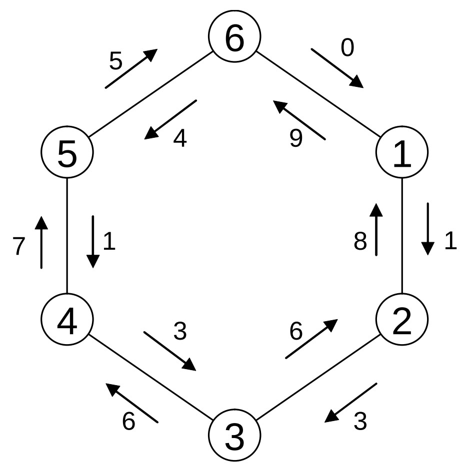
- $N = \frac{\lambda}{\mu-\lambda} = \frac{2.5}{5-2.5} = 1$
- $N_Q = N - \rho = 1 - 0.5 = 0.5$
- $T = \frac{1}{\mu-\lambda} = \frac{1}{5-2.5} = 1/2.5 = 0.4$ sec

Therefore, in total, $N = 10$, $N_Q = 5$, and $T = 0.4$ sec. Conclusion: packet switching provides better delay and queue length.

Problem 4: Max-Min fairness
Consider six nodes arranged in a ring and connected with unit capacity bidirectional links. There are two sessions from nodes 1, 2, 3, 4, and 5, to node 6, one in each direction. Similarly, there are two sessions from nodes 2, 3, and 4 to node 5, and two sessions from node 3 to node 4. Find the max-min fair rates for these sessions.

Hint: There are 9 sessions in each direction. Find how many sessions each link carries in each direction to determine which link must saturate first.

ANSWER:



The above figure shows the number of session on each link in each direction. There are 9 counterclockwise sessions and they all go through link (1,6). At step 1 this link is saturated, and the rate of all counterclockwise sessions is fixed at 1/9. At step 2 of the algorithm link (4,5) (the one that carries the most clockwise sessions) is saturated and the rate of the sessions going through it is fixed at 1/7. There are only two sessions (5 to 6) and (3 to 4 clockwise) that don't go through this link. At step 3 the link (3,4) becomes saturated and the rate of session 3 to 4 (clockwise) is fixed at 2/7. Finally, at step 4 of the algorithm link (5,6) becomes saturated and the rate of session 5 to 6 gets fixed at 3/7. The solution is now complete since the rate of all sessions has been fixed.