Introduction to the Theory of Computation
Homework 1
Due 9/8/2017
These are practice exercises

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Problem 0: Readings
(a) Read Chapter 0 in Sipser's book.

(b) Read Chapter 1 in Sipser's book.

Problem 1
The formal definition of a DFA M is $(\{q_1, q_2, q_3, q_4, q_5\}, \{u, d\}, \delta, q_3, \{q_3\})$, where δ is given by the following table:

	u	d
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_2	q_4
q_4	q_3	q_5
q_5	q_4	q_5

Give the state diagram of this machine.

Problem 2 (do as many as you want)
For each of the following languages, construct a DFA that accepts the language. Assume $\sum = \{0, 1\}$.

- $\{w|$ the length of w is divisible by three$\}$
- $\{w|w$ contains at least five $1s\}$
- $\{w|w$ contains the substring $1011\}$
- $\{w|w$ contains at least two $1s$ and at most two $0s\}$
- $\{w|w$ contains an odd number of $1s$ or exactly two $0s\}$

- $\{w|w$ begins with 1 and ends with $0\}$
- $\{w|$ every odd position in w is $1\}$
- $\{w|w$ has length at least 3 and its third symbol is $0\}$
- $\{\epsilon, 0\}$

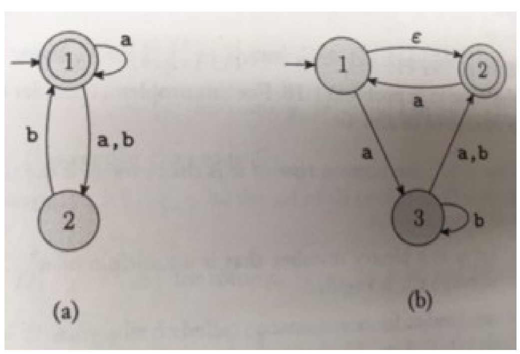
Problem 3 (do as many as you want)
For each of the following languages, construct an NFA, with the specified number of states, that accepts the language. Again, $\sum = \{0, 1\}$.

- $\{w|w$ ends with $10\}$ with 3 states
- $\{w|w$ contains the substring $1011\}$ with five states
- $\{w|w$ contains an odd number of $1s$ or exactly two $0s\}$ with six states
- $\{w|w$ contains the substring $11001\}$ with any number of states
- $\{w|w$ contains at least two $1s$ and does not end with $10\}$ with any number of states
- $\{w|w$ begins with 1 or ends with $0\}$ with any number of states

Problem 4
(a) Show that if M is a DFA that accepts language B, exchanging the accept and non-accept states in M yields a new DFA that accepts the complement of B, and conclude that the class of regular languages is closed under complement.

(b) Show, by giving a counter example, that if M is an NFA that accepts language C, exchanging the accept and non-accepts states of M does not necessarily yield a new NFA that accepts the complement of C. Is the class of languages accepted by NFAs closed under complement? Explain your answer.

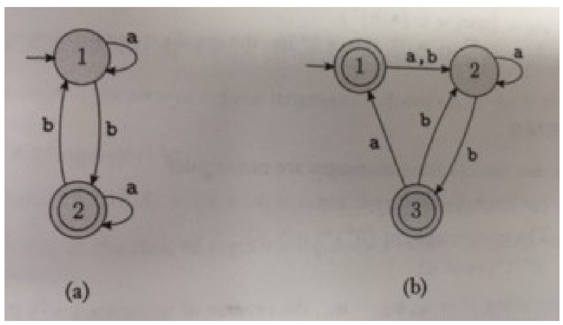
Problem 5
Convert the following NFAs to equivalent DFAs.



Problem 6
Convert the following regular expressions to NFAs.

- $(0 \cup 1)^*000(0 \cup 1)*$
- $(((00)^*(11)) \cup 01)*$
- $\emptyset*$

Problem 7
Convert the following finite automata to regular expressions.



Problem 8
Use the pumping lemma to show that the following languages are not regular:

- $\{0^n 1^n 2^n | n \geq 0\}$
- $\{ww | w \in \{a, b\}^*\}$
- $\{1^{2^n} | n \geq 0\}$