Introduction to the Theory of Computation
Homework 3
Due 9/26/2017
These are exercises

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Problem 0: Readings
Read Chapter 2 in Sipser's book.

Problem 1
Let $G = (V, \Sigma, R, S)$ be the following grammar. $V = \{S, T, U\}$; $\Sigma = \{0, \#\}$; and R is the following set of rules:

$S \rightarrow TT \mid U$
$T \rightarrow 0T \mid T0 \mid \#$
$U \rightarrow 0U00 \mid \#$

(a) Describe $L(G)$ in English.

(b) Prove that $L(G)$ is not regular.

Problem 2: CFL and complement
(a) Use the languages $A = \{a^m b^n c^n | m, n \geq 0\}$ and $B = \{a^n b^n c^m | m, n \geq 0\}$ to show that context-free languages are not closed under intersection.

(b) Use part (a) and DeMorgan's law (see Chapter 0) to show that the class of context-free languages is not closed under complementation.

Problem 3: CFG for some languages
Give context-free grammars for generating the following languages:

(a) The set of strings over the alphabat $\{a, b\}$ with twice as many a's as b's.

(b) The complement of the language $\{a^n b^n | n \geq 0\}$.

(c) $\{w\#x | w^R \text{ is a substring of } x \text{ for } w, x \in \{0,1\}^*\}$

(d) $\{x_1\#x_2\#\ldots\#x_k | k \geq 1, \text{ each } x_i \in \{a,b\}^*, \text{ and for some } i \text{ and } j, x_i = x_j^R\}$

Problem 4: Pumping Lemma for CFL
Use the pumping lemma to show that the following languages are not context-free.

(a) $\{0^n 1^n 0^n 1^n | n \geq 0\}$

(b) $\{0^n \# 0^{2n} \# 0^{3n} | n \geq 0\}$

(c) $\{w\#x | w \text{ is a substring of } x, \text{ where } w, x \in \{a,b\}^*\}$

(d) $\{x_1\#x_2\#\ldots\#x_k | k \geq 2, \text{ each } x_i \in \{a,b\}^*, \text{ and for some } i \neq j, x_i = x_j\}$

Problem 5: Chomsky normal form
Convert the following grammar to an equivalent one in Chomsky normal form using the procedure described in class.

$A \rightarrow BAB \mid B \mid \epsilon$
$B \rightarrow 00 \mid \epsilon$

Problem 6: Convert CFG to PDA
Convert the following context-free grammar to a pushdown automaton using the procedure described in class.

$E \rightarrow E + T \mid T$
$T \rightarrow T \times F \mid F$
$F \rightarrow (E) \mid a$