Introduction to the Theory of Computation
Homework 6
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Problem 0: Readings
Read Chapters 3 and 4 in Sipser's book.

Problem 1
Let $k-$PDA be a pushdown automaton that has k stacks. Thus a $0-$PDA is an NFA and a $1-$PDA is a conventional PDA. We already know that a $1-$PDA is more powerful (recognizes a larger class of languages) than a $0-$PDA.

(a) Show that $2-$PDAs are more powerful than $1-$PDAs by finding a language that can be recognized by a $2-$PDA but not by a $1-$PDA.

(b) Show that $3-PDA$s are not more powerful than $2-$PDAs. *Hint*: Simulate a Turing machine tape with two stacks.

Problem 2
A write-once Turing machine is a single-tape TM that can alter each tape square at most once (including the input portion of the tape). Show that this variant Turing machine model is equivalent to the ordinary Turing machine model. *Hint*: As a first step, consider the case whereby the Turing machine may alter each tape square at most twice. Use lots of tape.

Problem 3
A Turing machine with stay put instead of left is similar to an ordinary Turing machine except that the transition function has the form

$$\delta : Q \times T \rightarrow Q \times T \times \{R, S\}$$

At each point the machine can move its head right or let it stay in the same position.

(a) Show that this Turing machine variant is equivalent to a Turing machine that always moves its head to the right.

(b) What class of languages does this machine recognize?

Problem 4
Show that a language is decidable iff some enumerator enumerates the language in lexicographic order.

Problem 5
Let $A_{\epsilon CFG} = \{< G > \mid G \text{ is a CFG that generates } \epsilon\}$. Show that this language is decidable.

Problem 6
Let $INFINITE_{DFA} = \{< A > \mid A \text{ is a DFA and } L(A) \text{ is infinite}\}$. Show that this language is decidable.