Introduction to the Theory of Computation
Homework 8
Due 12/12/2017

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Problem 0: Readings
Read Chapter 5 and Chapter 7 (up to but not including NP-completeness) in Sipser's book.

Problem 1

(a) Let $J = \{w | w = 0x \text{ for some } x \in A_{TM} \text{ or } w = 1y \text{ for some } y \in \bar{A}_{TM}\}$. Show that neither J nor $\bar{J}$ is Turing-recognizable.

(b) Give an example of an undecidable language B, where B mapping reduces to $\bar{B}$.

Problem 2
A useless state is one that is never entered on any input string. Consider the problem of testing whether a useless state exists. Show that this problem is decidable for NFAs and PDAs and undecidable for Turing machines.

Problem 3
Problem 1: Test whether a Turing machine M on input w ever attempts to move its head left when its head is on the left-most tape cell.

Problem 2: Test whether a Turing machine M on input w ever attempts to move its head left at any point during its computations on w.

One of the above problems is decidable and the other is not. Which is which? Provide proofs.

Problem 4
Show that PCP is decidable when the alphabet $\Sigma = \{1\}$.

Problem 5
Let P be any problem about Turing machines that satisfy the following two properties:

- There exists two TMs M_1 and M_2, where $< M_1 > \in P$ and $< M_2 > \notin P$. In other words, P is nontrivial, it holds for some, but not all, TMs.
- For any TMs M_1 and M_2 where $L(M_1) = L(M_2)$, we have $< M_1 > \in P$ iff $< M_2 > \in P$. In other words, the membership of a TM M in P depends only on the language of M.

Show that P is undecidable.

Problem 6
Do exercises 7.1, 7.3, 7.8, 7.10, and 7.11.