Problem 1: Lattice points
We have five lattice points (points with integers as coordinates) in two-dimensional space. Show that the midpoint of one of the line segments that connect these points is also a lattice point. Hint: Each point has two coordinates each of which can be either even or odd.

Problem 2: Divisibility by $n$
Consider a list of $n + 1$ positive integers:

$a_1, a_2, \ldots, a_n, a_{n+1}$

(a) Prove that it is always possible to choose a pair of these whose difference is divisible by $n$. Hint: For each $a_i$, consider the remainder in the division by $n$.

(b) Suppose $a_{n+1}$ is now dropped from the list and $n > 2$. Prove that it is always possible to choose a pair whose sum or difference is divisible by $n$. Hint: What happens if you put $a_1, \ldots, a_n$ in $n$ boxes based on their remainder in the division by $n$?

Problem 3: Languages
In a certain class there are 25 students: 14 speak Spanish, 12 speak French, 6 speak French and Spanish, 5 speak German and Spanish, and 2 speak all three. The 6 that speak German all speak another language. How many speak no foreign language?

Problem 4: Card game
You are playing cards with 3 other players, call them Player 1, Player 2, and Player 3. You draw a 10. You will loose if any player gets J, Q, K, or A. What is the probability that you will loose? Hint: Say that a hand is good for Player $i$ if he gets J, Q, K, or A. Let $S_i$ be the set of hands that are good for Player $i$.

(a) Using Inclusion-Exclusion, find the number of good hands $|S_1 \cup S_2 \cup S_3|$. Dividing this by the total number of hands gives you the probability.

(b) Find the number of good hands in another way: First find the number of bad hands, then subtract it from the total number of hands.