will take hourly readings of the barometer, thermometer, etc. The readings are to start 6 hours before noon on the 21st, taking the whole day, and continuing until 6 hours after noon of the 22nd. If the 21st is a Sunday, they will not begin until the 22nd. It is important that the same days are chosen at different parts of the earth.

The persons who will take part in these observations can send their results to Sir J. Herschel or M. Babbage, using the Académie as intermediary.

IV

ON THE MATHEMATICAL POWERS OF THE CALCULATING ENGINE[a]

(26 December, 1837)

The object of the present volume is to show the degree of assistance which mathematical science is capable of receiving from mechanism. I may possibly in a separate work describe minutely the mechanism I have contrived for that purpose. For the complete understanding of the following pages it will be unnecessary to present to the reader more than a very general outline of the structure of the engine and if he feel indisposed to examine even that, he may pass it over and taking certain mechanical data for granted at once proceed to the mathematical investigation in which he will find that they are all proved to depend on those mechanical data.

The calculating part of the engine may be divided in two portions:

First. The *mill* in which all operations are performed

Secondly. The *store* in which all the numbers are originally placed and to which the numbers computed by the engine are returned.

The plate[b] represents a plan of the engine – those circles placed round

[a] 'On the mathematical powers of the calculating engine' is a manuscript in the Buxton MSS collection in the Museum of the History of Science, Oxford. The manuscript was given by Babbage to his friend H. W. Buxton, who quoted it at length in his *Memoirs of the life and labours of the late Charles Babbage* (first published 1987). The complete manuscript was first published in B. Randell, *The origins of digital computers* (Berlin etc., 1973). This version has been lightly edited for readability: the numbering of sections has been made uniform; rules (__) have been used to indicate missing values, and ellipses have been used to indicate unfinished passages in the original. All substantive corrections have been indicated by footnotes.

[b] No plate accompanies the manuscript, but it is likely that Babbage intended to use the general plan, no. 25 dated 6 August, 1840, reproduced as the final plate of the appendix of this volume.

the great central wheel constitute the mill whilst that portion which adjoins the longitudinal part or rack represents the store.

I OF THE MILL

Two axes with figure wheels, **I** the *ingress axis* and ″**A** the *egress axis* connect the *mill* with the *store*.

The mill itself consists of:

1. three figure axes
2. three carriage axes
3. ten table figure axes
4. digit counting apparatus
5. selecting apparatus
6. barrels
7. reducing apparatus for barrels
8. operation cards
9. repeating apparatus
10. combinatorial counting apparatus.

1. *Of the figure axes*

The figure axes **A** and ′**A** are connected with each other without the intervention of the central wheels so that a number on the figure wheels of one axis may be transferred to those of the other.

These figure wheels are considerably larger than any others in order to allow of sufficient space on their circumference for placing the pinions by which communications are made with other parts of the mill.

By means of some of these pinions a process called *stepping down* and another called *stepping up* may be performed. It consists in shifting each digit of a number one cage lower or one cage higher, which processes are equivalent to the arithmetical operations of dividing or multiplying the number by ten.

Other pinions are fixed on *register axes* **R** and $\mathbf{R}_1$ and convey the two highest figures of the dividend to the *selecting* apparatus. The third figure axis ″**A** is placed near the store and constitutes the egress axis. It is adjacent to the digit counting apparatus with which it communicates.

2. *Carriage axes*

These axes **F**, ′**F**, ″**F** with their peculiar apparatus are employed to execute the carriage of the tens when numbers are added to or subtracted from each other. The carriages **F** and ′**F** can be both connected with the figure axis **A** or one of them with the figure axis **A** and the other with ′**A** or they may by means of the central wheels be connected with any other part of the mill. The third carriage ″**F** is connected both with the mill and the store and may be used with either.

Whenever the number subtracted is greater than that from which it is taken the resulting carriages would if effected, and if the mechanism admitted, produce a carriage in the forty-first cage. This fact is taken advantage of for many purposes – it is one of very great importance and when it happens a *running up* is said to occur. Connected with this part is a lever on which the *running up* warning acts and this lever governs many parts of the engine according as the circumstances demand.

3. *Table figure axes*

These axes are ten in number, nine of them contain the table of the nine multiples of one factor in multiplication and of the divisor in division. The tenth contains the complement of the divisor in the latter operation. They are all connected with the central wheels and the number on each figure wheel can be *stepped up* or *down* upon the other figure wheel of the same cage. The figure which at each stepping goes off from the bottom wheel is transferred to the top wheel.

4. *Of the digit counting apparatus*

This is a mechanism by which the digits of any number brought into the mill may be counted and certain calculations made as to the position of the decimal point in the result of multiplication and division. It is also used to limit the number of figures employed when the engine is making successive approximations, either to the roots of equations or to the values of certain functions. It consists of three distinct systems nearly similar to each other.

5. *Of the selecting apparatus*

When a table of the nine multiples of a multiplicand has been made it becomes necessary in order to effect multiplication to select successively those multiples indicated by the successive digits of the multiplier. This mechanically is not difficult. But when in the process of division it becomes requisite to select that multiple which is next less than the dividend from which it is to be subtracted the mechanical difficulty is of quite a different order and hitherto nothing but the most refined artifices have been found for accomplishing it. This refinement relates however entirely to the *nature* of those contrivances not to the certainty of their action nor to any delicacy of workmanship.

The apparatus consists of a portion of the carrying apparatus for three figure wheels which by the addition of another contrivance renders them available for the purpose of making the selection. This apparatus is placed immediately below the table axes.

6. *Of the barrels*

The barrels are upright cylinders divided into about seventy rings, the circumference of each ring being divided into about eighty parts. A stud may be fixed on any one or more of these portions of each ring. Thus each barrel presents about eighty vertical columns every one of which contains a different combination of fixed studs.

These barrels have two movements:

1. They can advance horizontally by a parallel motion of their axis.
2. They can turn in either direction and to any extent on their axis.

When the barrels advance horizontally these studs act on levers which cause various movements in the mill, the stud belonging to each ring giving a different order.

Amongst these movements or rather these orders for movements the following may be more particularly noticed. The advance of a barrel may order:

1. A number with its sign to be received into the mill from the ingress axis.
2. A number with its sign to be given off from the mill. This number may thus be either altogether obliterated from the mill or it may at the same time be received on the egress wheel or the number may be given

off from the mill to the egress wheel and at the same time be itself retained in the mill.
3. A variable card to be turned.
4. An operation card to be turned.
5. The circular movement of the barrel itself or of any other barrel to another vertical. This always occurs at every step from the beginning to the end of what are called *operations*. The barrels when once ordered by the operation cards from their zero point to any given vertical always direct themselves to be turned to another vertical preparatory to their next advance. This circular motion is however occasionally changed by an action arising from another source.

7. *Of the reducing apparatus*

Behind each barrel is placed a reducing apparatus. It consists of six or eight sectors which can be made to act upon the barrel and give it a rotatory movement so as to make it pass over 1, 2, 3, or any required number of verticals previously to its next advance.

The levers which put these sectors into action are acted upon by:

1. the studs on their own barrel
2. the studs on any other barrel
3. the operating cards
4. the running up levers.

The first and third of these sources of action occur most frequently.

8. *Of the operation cards*

Those who are acquainted with the cards of a Jacquard's loom will readily understand the functions performed by these cards. To those who are unacquainted with that beautiful contrivance it may be necessary to state that the *cards* consist of pieces of thick pasteboard, tin plate, or sheet zinc pierced with a number of holes; these cards being strung together by wire or tape hinges pass over a square prism.

This prism is situated in front of a number of levers placed in rows which govern the reducing apparatus and consequently the barrels. The faces of the prism are perforated so as to present an opening opposite *every* lever.

If the prism alone is made to advance horizontally against these levers then the levers themselves will enter into the holes of the prism and be partly covered by it but they will not be moved out of their places.

Again if a card having as many holes as the prism has, or as there are levers opposite to it, is placed upon the advancing face of the prism no effect can be produced on the levers by this advance of the prism. But if a card having one hole less than the prism is placed on its face then when the prism advances the lever opposite that hole will be pushed and any order given for which that lever was appointed. Suppose after every order the levers to be replaced and let the prism be turned one quarter round; then a new card will be presented to the levers and if one or more holes of this second card are stopped up a different order will be transmitted through the levers to the reducing apparatus and thence to the barrels.

Thus by arranging a string of cards with properly prepared holes any series of orders however arbitrary and however extensive may be given through the intervention of these levers.

The number of the levers acted upon by the operation card is small; they respectively direct the barrels to *commence* the following operations:

1. the *addition* of two numbers
2. the *subtraction* of one number from another
3. the *multiplication* of two numbers
4. the same *multiplication limited* to a given number of the first figures
5. the *division* of one number by another
6. the same *division limited* to a given number of figures in the quotient.

The levers numbered 4 and 6 are rarely used; the extraction of roots being the only case in which they are required.

These cards are called into action by orders from the barrels. What they shall order when acting depends on the nature of each individual card. What repetitions they shall be subject to depends on the orders communicated to them from the combinatorial cards and their counting apparatus. Many calculations are much simplified by having two sets of variable cards.

9. *Of the combinatorial cards*

One or more peculiar cards may be inserted amongst the operation cards of certain formulae. They are called *combinatorial cards.* The object of these cards is:

To govern the *repeating apparatus* of the operation and of the variable cards and thus to direct at certain intervals the return of those cards to given places and to direct the number and nature of the repetitions which are to be made by those cards.

Whenever combinatorial cards are used other cards called index cards must occur amongst those of the formulae. The use of these cards is to compute the numbers which are to serve successively for the indices of the combinatorial cards. At what time the combinatorial cards shall act depends on the number of repetitions the last of those cards appointed. What orders each combinatorial card shall give depends on the nature of each individual card.

II OF THE STORE

The store may be considered as the place of deposit in which the numbers and quantities given by the conditions of the question are originally placed, in which all the intermediate results are provisionally preserved and in which at the termination all the required results are found.

Various parts may be added to the store according to the purposes required. Some of them might perhaps with more convenience constitute distinct machines.

The store then may contain:

1. figure axes
2. computing apparatus
3. number cards
4. card punching apparatus
5. printing apparatus
6. copper punching apparatus
7. curve drawing apparatus
8. variable cards.

The figure axes and the variable cards alone are absolutely necessary for the mathematical enquiries in the present work and for the sake of simplicity the others will be only occasionally referred to.

1. *Of the figure axes*

A number of axes each having forty figure wheels placed in different cages one above another are connected with the rack of the store.

These figure wheels are each numbered from 0 to 9; they may be turned by hand so that any digit may stand opposite a fixed index. Thus any number of not more than forty places of figures may be put upon the figure wheels of each axis.

Above the fortieth cage is another cage containing a wheel similar to a figure wheel and also having its circumference divided into ten parts. These parts have the signs (+) plus and (−) minus alternately engraved upon them. Above this wheel is a fixed character to distinguish each particular axis or rather the variable number which may be found upon

	Variables of the Engine	V_1	V_2	V_3	V_4	V_5	V_6
Signs	Cage 41	+	−	+	−	+	+
Numerical coefficients	Cage 40	0	0	0	0	0	0
	Cage 39	0	0	0	0	0	0
	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	Cage 5 Tens of thousands	0	0	0	2	0	0
	Cage 4 Thousands	0	1	4	3	0	0
	Cage 3 Hundreds	0	7	9	4	0	2
	Cage 2 Tens	0	5	7	1	3	4
	Cage 1 Units	0	8	1	0	6	7
	Variables of the question		a	x	x^3	$\sin\theta$	$\sqrt{a^2+x^2}$

its wheels. These fixed marks are V_1, V_2, V_3, ... as far as the number of quantities which can be contained in the store.

Below the lowest or units figure wheel a small square frame appears in which may be inserted a card to be changed according to the nature of the calculation directed. On this card is written that particular variable or constant of the formula to be computed whose numerical coefficient and sign are expressed on the wheels above it.

The annexed representation will perhaps convey a clearer idea of this part of the engine.

The first line of quantities V_1, V_2, ... are never altered; they merely indicate the particular sets of figure wheels to which they are attached.

The next line contains the *signs* of the quantities which are themselves expressed by writing them upon pieces of card and placing them in the squares at the bottom.

The intermediate forty cages contain the numerical coefficients.

The first variable or V_1 is in the present figure equal to zero and previously to the setting of the engine to any problem; all the other variables V_2, V_3, V_4, ... have the same symbols beneath them.

The second variable V_2 has beneath it the negative sign, the number 1758 and the algebraic quantity a.

The symbols placed on the engine are in the annexed figure:

$V_1 = 0,$

$V_2 = -1758,$ $\quad a$

$V_3 = +4971,$ $\quad x$

$V_4 = -23410,$ $\quad x^3$

$V_5 = +36,$ $\quad \sin\theta$

$V_6 = +247,$ $\quad \sqrt{a^2 + x^2}$

In this manner any function however complicated it may be, if it is considered as a whole, may be placed in the store with its proper sign and its numerical coefficient; the function itself being merely written on a card and placed in the square below its coefficient. The number of variables which can be contained within the store will depend on the length of the rack and number of figure axes which can be placed round it, and although a large number of variables might with perfect safety be employed yet there is obviously a practical limit arising from the weight of the rack to be moved.

One hundred variables would not give an inconveniently large rack but still the calculations of such an engine would be limited. This limitation can be entirely removed by another set of cards called number cards which will presently be described.

If any of the coefficients contain decimals or if the result is required with any number of decimals then all the coefficients must be considered as having the same number of decimals. If an imaginary line is drawn between any two cages, the third and fourth for example, then all below it may be considered as decimals. In order to convey to the engine this information there exists a wheel with the numbers from 1 to 40 engraved on its edge; this wheel being set at any number the engine will treat all the numbers put into the store as having that number of decimals.

2. *Of the computing apparatus*

One of the figure axes **K** has its figure wheel connected with the rack differently from the others. It is also connected with the carrying axis ″**F** belonging to the mill from which however it may at times be disconnected.

The object of this is to enable the figure wheel on the rack to make certain simple calculations without the necessity of sending the numbers into the mill. During the process of one of the great operations in the mill a series of numbers may be computed in the store by the method of differences and thus a considerable saving of time effected.

3. *Of the number cards*

The number cards have been introduced for the purpose of rendering the calculations absolutely unlimited by the too great number of variables and constants necessary for the solution of any problem.

The number cards are pierced with certain holes and stand opposite levers connected with a set of figure wheels placed on the number axis which can be made at intervals to communicate with the rack. When these number cards are advanced they push in those levers opposite to which there are no holes on the cards and thus transfer that number together with its sign which the holes on the card represent to the figure wheels of the axis opposite to which the cards are placed. The number

and its sign thus put upon these figure wheels may be immediately transferred to another part of the store, and the string of number cards being turned the next card conveys its number to the store in the same way.

These number cards are for some purposes more convenient than figure wheels because the numbers upon a figure card are not obliterated by the act of giving them off. For by turning the string of figure cards back to any given one the number upon that card can be replaced in the store as frequently as may be required and at any periods of time which the calculation may demand. On the other hand the numbers placed upon figure wheels are always obliterated in the act of giving them off. If it is necessary to retain on the store any number which is to be given off to the mill then it also must be given off through the rack to another store axis on whose figure wheels it must remain until at a second operation it is reconveyed by rack back to its original place.

4. *Card punching apparatus*

One mode of rendering permanent the results of any calculations made by the engine will obviously be by making it punch on cards certain holes similar to those just described as existing on the number cards. This plan will also enable us to use any intermediate computation which may be necessary in advancing towards the final results, for the cards so made may from time to time be removed from the punching apparatus and attached to the number cards. Other advantages will be observed when the subject of mathematical tables comes under consideration.

5. *Printing apparatus*

It is desirable even when many copies of a calculation made by the engine are not wanted that the results should be themselves printed by the machine in order to ensure the absence of error from copying its answers.

A set of thin circular rings having metal types of the digits fixed at equal distances on their edges and themselves governed by the calculating wheels on which the result is placed are to be pressed down at intervals on a sheet of paper covered by another sheet of carbonized paper. This paper is fixed in a platform having proper motions for placing the printed results in right order.

Thus a single and correct copy may be produced although from the nature of the process the execution of the printing would not be of the highest order.

If however many printed copies are required then it is intended that the type so arranged shall be made to impress their characters on a soft substance from which mould a stereotype plate may be cast.

6. *Copperplate punching apparatus*

If it should be deemed necessary to print tables or calculations upon copperplate an apparatus has been contrived for that purpose. This process is necessarily slower in its operation then the former modes of rendering the calculated results permanent. It has however the advantage of possessing greater clearness although the additional cost of taking off impression may in some instances be objectionable.

Since, however, the invention of the number cards these modes of printing or engraving have ceased to become essential parts of a calculating engine. The absolute certainty of every printed result can now be obtained although the printing mechanism be totally detached from the calculating portion of the engine, an improvement which it was impossible to make until that point of the enquiry was attained.

The cards on which the results are punched may themselves be placed in a distinct machine and from the holes formed in them the new machine may either engrave or print them as it may have been prepared to operate.

7. *Curve drawing apparatus*

The discovery of laws from the examination of a multitude of tabulated and reduced observations is greatly assisted by the representation of such tables in the form of curves.

As one of the employments of a calculating engine would be to reduce

collections of facts by some common formula, I thought that at the time it impressed the computed results it would be desirable that it should mark the point of a corresponding curve upon paper or copper if preferred. The three or four first figures of the table will be expressed by the curve. The contrivances for this purpose are not difficult and their employment does not lengthen the time of the calculation.

8. *Variable cards*

The variable cards are appointed for the government of the various parts which constitute the store. Like all the other cards they act by pushing forward certain levers placed in front of them. These levers cause the motions of the several parts of the store which have been described.

It is necessary in the present work to consider only: the figure axes; the number, variable, combinatorial, and operation cards; and the card punching apparatus.[a]

With respect to these, the principle functions of the variable cards will be to direct:

1. A number and its sign to be given off from any store axis to the ingress axis.
2. A number and its sign to be received upon any store axis.
3. Any number and its sign to be given off from the number cards to the wheels on the number axis.
4. Any number and its sign to be given off to the card punching apparatus and a corresponding card to be punched.

The number of levers necessary for these purposes is not so large as might at first appear, consequently the cards need not approach an inconvenient magnitude. For example fourteen levers and their equivalent fourteen holes will be all that is required in the third of the above divisions for eight thousand variables.

If the other appendages to the store which have been already described should be thought necessary a small number of additional levers must be added.

[a] This paragraph has been reworded to incorporate an addendum to the manuscript.

III OF OPERATIONS

The operations which can be performed in the mill are the following:

addition
subtraction
multiplication
division
extraction of roots.

These five processes are all which it will be necessary to consider in the present volume. Others might be added if required and the additional mechanism would be small, new barrels with studs differently arranged would be nearly the whole change.

If these operations are merely viewed in an arithmetical light the engine would greatly assist us in the numerical calculations to which analysis leads, but it possesses the power of treating the signs of the quantities on which it operates according to the rules of algebra and thus its use is greatly extended.

1. *Addition*

There are three principles on which mechanical modes of adding numbers together may be constructed.

An axis passing through a wheel whose circumference is divided into ten parts may be conceived at certain periods to make a revolution independently of the wheel. This axis therefore will, whenever it moves, have an angular motion of one circumference which is equal to ten divisions on the wheel.

Now if a stud projects from the upper surface of the wheel and if a fixed arm project from the axis in the same plane, then whenever the axis revolves its arm will at some period in its revolution come into contact with the stud on the wheel and will carry it forward until the axis stops.

If the wheel had been so placed previously to the movement of the axis that the stud upon the wheel stood *one* division in advance of the arm then the wheel would have been driven over nine divisions; if the wheel had been so placed at the commencement that the stud had been *six* divisions in advance of the arm then it would have been moved over four divisions. In the first case the number *nine* would have been added

to the wheel and in the latter case the number *four*. This addition might be communicated by a pinion to any other wheel.

Thus by rightly placing the first wheel, any number from one to nine may be added to a second wheel. When such mechanism is employed the axis moves uniformly round and picks up the wheel at a time dependent on the digit it is to add. Also the motions of the wheel and of the axis terminate together.

Another method of adding might be imagined in which the axis should be bolted to the wheel at the commencement of its motion and carry it along for a certain part of its motion only, some subsidiary mechanism unbolting it when the wheel had reached a certain division.

In this case whatever be the number added the motion of the wheel commences with that of the axis but ends at different periods according to the nature of the number added. The combination of these two principles produces a third mode of adding in which neither the beginning nor the end of the additions occur at fixed points.

The three principles then by which addition may be executed mechanically may be thus stated:

1. By the movement of a wheel which always ends its motion at a definite time independently of the particular number to be added but which commences its motion at a time determined by that number.

2. By the movement of a wheel which always commences its motion at a definite time but ends it at a period dependent on the particular number added.

3. By a wheel both commencing and terminating its movement at periods dependent on the number added.

The first of these principles is adopted in the present engine. The second exists in the difference machine. The third presents greater difficulties in the execution without equivalent advantages.

In all these plans the number placed on the first wheel and which is to be added to some other number is itself obliterated by the operation. They might therefore with propriety be called apparatus for reducing figure wheels to zero. Another class of contrivances has been drawn in which the number added is itself retained so that the same number may be repeatedly added. The zero apparatus may generally be converted into an adding and retaining apparatus by the addition of two other wheels.

The process of addition requires for its completion that an apparatus should be attached to the wheels on which the sum is placed for the purpose of carrying any tens that may occur to the next highest digit.

This mechanism may be constructed upon three different principles. It may be: successive in its operation; it may postpone its operations; it may anticipate operations.

Carriage may be successive in its operations. Those of the first class are the most simple but the time necessary for the performance is an effective impediment to their employment in engines like that we are considering. Let us suppose unity to be added to 999999 then

Add together	1
	00999999
Time of 1st carriage	00999990
	1
Time of 2nd carriage	00999900
	1
Time of 3rd carriage	00999000
	1
Time of 4th carriage	00990000
	1
Time of 5th carriage	00900000
	1
Time of 6th carriage	00000000
	1
	01000000

The time necessary for the mere addition and leaving a notice that the first figure in the right has passed a ten (which is indicated by putting the unity below the tens figure) must be at least equal to that which is required for the figure wheel to pass over nine divisions. Now the time for adding the first ten which is carried and giving notice of the second will be equal to one division and so on, so that at the end of the sixth carriage a time equal to fifteen divisions will have been passed over and it is to be observed that another division must be allowed because the machine cannot know that there is not another carriage due to the seventh figure.

Thus in an engine having forty figures we should have for every addition and its carriage

for addition	9
for 39 possible carriages	39
	48

So that in fact the time consumed in making the carriage of the tens would be more than four times as much as that required for addition. Modes have been contrived for shortening this time but it is evident there is a limit beyond which it cannot be reduced.

Carriage may postpone its operations. When many additions are to be made to the same quantity, as in multiplication, time might be saved by reserving the notices of carriage and executing them altogether.

Carriage may anticipate its operations. If the mechanism which carries could be made to foresee* that its own carriage of a ten to the digit above when that digit happens to be a nine would at the next step give notice of a new carriage then a contrivance might be made by which, acting on that knowledge, it should effect both carriages at once. Thus, after the addition of the numbers below

Time of addition	123456
	346601
Time of 1st carriage	469057
	1
Time of 2nd carriage	460057
	1
	470057

* In substituting mechanism for the performance of operations hitherto executed by intellectual labour it is continually necessary to speak of contrivances by which certain alterations in parts of the machine enable it to execute or refrain from executing particular functions. The analogy between these acts and the operations of mind almost forced upon me the figurative employment of the same terms. They were found at once convenient and expressive and I prefer continuing their use rather than substituting lengthened circumlocutions.

For instance, the expression 'the engine *knows*, etc.' means that one out of many possible results of its calculations has happened and that a certain change in its arrangement has taken place by which it is compelled to carry on the next computation in a certain appointed way.

If at the time of the first carriage the engine knew the next number to which it was about to carry was a nine and that consequently it would *afterwards* become necessary to give notice of and to carry another ten then it might be taught to execute both these carriages at the same time and consequently to anticipate the time of the second carriage.

This object is of such importance that it is worthy of any labour to obtain a plan at once simple and effective. I have contrived several; that which is adopted in the present engine is perfectly secure and considering the very complicated nature of the conditions it is not very deficient in simplicity. This extreme complication of the conditions will perhaps be more felt by considering the following cases[a]

```
Time of addition     006012345678
                     001987655022
                     ------------
                     007999990690
Time of carriage            1  1
                     ------------
                     008000000700

Time of addition     0004365780064128976438052
                     0005634238990771023562048
                     -------------------------
                     0009999918954899999990090
                            1  1         1  1
                     -------------------------
                     0010000019054900000000100
```

The complexity arises from the circumstance that the result of any carriage *may* effect the value of *every* figure on its left hand and that the effect of *any* carriage may terminate at any digit and a *new* carriage may commence its effect at the succeeding one; and this like its predecessor may or may not be the precursor of many others.

2. *Of the algebraic signs as used in addition*

It has been already stated that above every set of forty figure wheels there is in the 41st cage a sign wheel and also that this sign wheel is divided like the figure wheels into a number of parts which is equal to

[a] The diagram has been corrected.

some multiple of ten, the sign minus being engraved on the odd, the sign plus on the even divisions.

Now when these wheels all stand at zero the sign wheel being also zero will present the positive sign.

When a positive number is added its sign being fixed on some even number the sign wheel of the ingress axis will be advanced by an even number of digits and must consequently present some other even number on which will appear the sign *plus*.

The ingress wheels of the mill which receive any number receive also its sign, but as the number on the ingress wheels was zero and the sign plus, the addition merely causes the ingress wheels to stand at the number entering and the sign to remain as it was before, namely at plus. For the 41st wheel stood at + or at an even number, the addition of + or any other even number will produce an even number on this wheel and consequently it will still present the sign plus. If on the other hand the number entering has the negative sign then its sign wheel stands at an odd division and as an odd number added to an even one produces an odd number the sign of the number in the ingress wheels will be negative.

If therefore when the first of two quantities, which it is proposed to add together, is placed upon the ingress wheels the sign wheel stands at an even number, *no change* is made on the barrels; but if, when the sign of that quantity is negative or the sign wheel of the ingress axis stands at an odd division, a *certain change* is made, then the mill may be said to *know* that a negative quantity has entered and the barrels may give the orders necessary in consequence.

If two numbers P and Q are to be added together the result will be

$$+P+Q$$

where the two signs + + may be called the algebraic signs; but if both P and Q are the result of certain preliminary computations made in the engine it may not be known whether the numbers themselves denoted by P and Q are positive or negative. In this case we must distinguish between the known algebraic signs and the possible or contingent signs resulting from the computation – these latter may be called the *accidental signs*. The above formulae may therefore be written thus

$$+(\pm P)+(\pm Q)$$

The process of addition or subtraction is thus executed:

An operation card advances and orders the variable cards to advance;

in the next turn a variable card containing the order for the sign plus or minus advances and places the sign on the sign wheels of ″**F** whilst at the same time it had ordered the number P to be given off from the store to **I**. The sign on the card may be called the algebraic sign. When the number comes in from the store it will bring with it its own proper sign which will depend on the previous operations by which it had been formed, and it will be convenient to call this latter its accidental sign.

This accidental sign will be added to the algebraic sign upon ″**F** and thus the two signs will be united and we shall have $\pm(\pm P)$ reduced to $\pm P$, any combination of the above signs being used.

If algebraic and accidental signs are of the same kind P is added to the figure wheels on **A** which have a positive sign on them. If the algebraic and accidental signs are different P is subtracted from the wheel on **A** and in consequence of a running up the negative sign is put upon the sign wheel of **A**.

The same process takes place with respect to $\pm(\pm Q)$ which after the combination of its two signs is transferred either additively or subtractively to the same wheels of **A**. In this second case a running up may or may not take place; if it does the previous sign is reversed.

The sum of two numbers P and Q as modified by their respective signs will now be found on the figure wheels of **A** either in the form of a number or its complement.

If it is number it and its sign will be transferred to ″**A**.

If it is complement by being subtracted from ″**A** it will appear as number and a negative sign will appear on the sign wheel.

This being premised the process of addition may be thus stated:

At the first turn of the handle an operation card is turned; it advances and acting on the barrels through the reducing apparatus orders them to move to that vertical which commences addition. It also orders a variable card to advance. At the next turn of the handle the barrels advance and direct the ingress figure wheels to receive from the store any number and its sign which the variable cards may have ordered to be given off. The barrels in advancing order themselves to turn circularly after their advance so as to present the next vertical necessary for performing addition. If the *accidental sign* of the number on the ingress wheels is positive this arrangement continues, but if that sign is negative it effects a change in the reducing apparatus and before the next turn the barrels have been moved to a different vertical which gives the orders necessary from the fact of the accidental sign of the number brought in being negative.

At the third turn the barrels advance and order the figure wheels on **A** and on the two carriages **F**, ′**F** to receive the number and its sign, both duly modified, from the ingress figure wheels whilst at the same time the variable card which was turned in the last turn of the handle orders a new quantity Q and its accidental sign out of the store onto the other figure wheels of the ingress axis. But this second advance of the barrels has ordered a circular motion for themselves by which they would turn to the next vertical for addition whilst the *accidental sign* of the quantity Q introduced on the ingress wheels again modifies this order if it happens to be negative.

. . .

3. *Subtraction*

After the explanation of the process of addition with the accidental sign of the quantities added together it will not be difficult to understand that the operation of subtraction is carried on in a similar manner. As in addition the subtraction of a greater number from a less (in that case arising from the accidental signs) causes a running up which gives notice that the resulting quantity is the complement of a negative number; so in subtraction where the algebraic sign produces the same result a similar effect follows. The notations which follow will explain more clearly the several stages of the operation as well as the parts taken in it by the several portions of the mill.

4. *Of the time required for the operations of addition and subtraction*

Any two numbers P and Q whose sum does not exceed forty places of figures may be placed in the engine and whatever be the nature of their accidental signs they may be brought out of the store placed in the mill and the operation of addition or subtraction performed as indicated by the formula

$$\pm(\pm P) \pm (\pm Q)$$

and the result may be returned into the store in six turns of the handle. The first of these turns might take place as the final turn of any previous operation so that the time really required is six turns of the handle. If another quantity $(\pm R)$ is to be added or subtracted it will require another turn and generally if n quantities with their accidental signs are to be added and subtracted the time required for the operations will be $n + 4$ turns of the first mover.

The time which the edge of the figure wheel occupies to pass from one figure to the next figure is considered as the unit of time and the space between two adjacent figures is the unit of space. The cases in which that velocity is exceeded are very rare in any part of the machine and only occur when the moving part is carried on to a fixed stop. Wishing to under- rather than overrate the speed of its calculations I have assumed that the velocity of the circumference of the figure wheel when moving will be ten feet or one hundred and twenty inches per minute. A single turn of the first mover will according to circumstances be performed either in fifteen or in twenty of such units. In the process of addition cycles of twenty units only are employed and as each unit requires ·15708 seconds the addition of two numbers only will require __ seconds and the addition of n quantities will cost 21·99 or nearly 22 seconds.

5. *Multiplication*

The process of multiplication is thus performed by the engine:

The two factors P and Q with their respective accidental signs being placed on the two sets of figure wheels No. 1 and No. 2 of the ingress axis, an operation card is turned. This directs the barrels through the reducing apparatus to move circularly to the vertical which commences multiplication.

At the next turn the barrels advance and order the reception of one of the factors Q. They also direct themselves to move on their axes to the second vertical belonging to multiplication.

At the third turn the barrels advance and order the reception of the other factor P from the second set of ingress wheels. At this turn the factor P is subtracted from the factor Q which had at the preceeding turn been placed upon the figure wheels of 'F. The result of this subtraction determines which of the two factors is largest and consequently which factor is to be tabulated.

Much time is saved by this decision for supposing one factor to contain only four places of figures and the other to contain thirty five figures. Then if a table of the first nine multiples of the first factor were made its multiplication by the other factor would require thirty-five-additions whilst if the larger factor had been tabulated only four

additions would have been necessary. The mechanical mode by which this knowledge is conveyed to the barrels is thus. The barrels, after ordering the subtraction of P from Q in the axis 'F, direct themselves to move on to another vertical; if P is less than Q no *running up* takes place and the order thus given by the barrels is obeyed; but if P is larger than Q then a *running up* takes place and the order given by the barrels to move to a certain vertical is enlarged and they are directed by the running up lever to move on to a different vertical. This new vertical directs the tabulation of the second factor whilst the former vertical appointed that of the first factor.

At the next turn the number which the mill has decided on tabulating is added to each of the nine table axes and to the carriage **F**.

At the next turn the same number is added to eight of the table axes and the carriage axis, then to seven, and so on until the nine multiples have all been formed in the nine table axes.

During these turns the number of the digits in the multiplier has been ascertained and placed in the counting apparatus No. 1.

The multiplier having been prepared is now stepped down and acts upon the *selecting* apparatus. Whatever digit exists in the lowest figure of the multiplier, by means of the selecting apparatus the corresponding multiple is added over to the product axis '**A**. During this addition each multiple is stepped up one cage higher in its own figure wheels; also the multiplier being stepped down its next figure is conveyed to the selecting apparatus preparatory to the next addition to the product.

The product is thus formed beginning with its lowest figure on the figure wheels of the axis **A**, but in the progress of stepping up the highest figure on the table axes may at length reach the top or 40th cage of their axes. At the next stepping the highest figure would of course be stepped off the machine and lost but this is prevented by a communication with the lowest figure wheel on the same axis to which the number stepped off from the highest is transferred. These figures which are so transferred to the lower cages are not added to the same axis but are by a particular mechanism added over to a set of figure wheels on the axis **A** and the carriage **F** on which the head of the products is formed.

If a number of twenty figures only is multiplied by another factor containing an equal number of digits, as the product will not exceed forty digits it will be contained on the first axis. But if both factors contain forty digits then the head or first forty digits of the product will be formed on the axis **A** and the tail or second forty digits of the products will be formed upon '**A**.

The signs which entered with the original factors, having been added together on the sign wheels, will give the resulting sign of the product. For if they were both positive then each sign being equivalent to an even number, the sum of the two signs would give an even number or its equivalent the positive sign. If on the other hand both signs were negative then a negative sign being equivalent to an odd number, the sum of two odd produces an even number which also is equivalent to the positive sign which the product ought in this case to possess. Lastly, if the sign of one factor be positive and that of the other negative, then the sum of an even and of an odd number being odd, the negative sign results.

The termination of multiplication arises from the action of the counting apparatus which at a certain time directs the barrels to order the product thus obtained to be stepped down so that the decimal point may be in its proper place, and then to turn to another vertical, the consequence of which is the transfer of the product with its proper sign to the wheels on the egress axis from whence they may be removed by variable cards and the barrels to any axes in the store.

The great difficulties in constructing the calculating engine have in general arisen from certain conditions I had laid down as desirable for the perfect security and certainty of its action rather than from the difficulty of inventing mechanical contrivances for performing the functions of its several parts. There are few of its actions which have not been contrived and in many instances drawn according to several different principles. The only exceptions I should be inclined to make as to this relative difficulty are perhaps the mode of carrying the tens by anticipation and amongst the operations the mode of dividing one number by another.

The following principles were proposed:

No movement should depend upon the action of a spring.

This condition placed great difficulties in the way of many contrivances and the modes of obviating them are very various. It is not however proposed to exclude *all* springs, for those which merely retain a lever or a wheel that has been already brought to its exact place by other means are unexceptionable. In one sense they relieve the machine from the continual drag to which stiff friction if substituted would expose it, and on the other they save a multitude of movable axes which would otherwise become necessary and they also enable wheels to remain retained until certain indefinite actions take place.

Another principle is that:

Every movement shall be of such a kind that the engine shall either break itself or stop itself or execute the intended motion.

A third principle is:

That the several operations shall be executed in the shortest possible time.

This last principle has caused greater difficulties than either of the former – in fact the whole history of the invention has been a struggle against time. As soon as any contrivance has been made which was unexceptionable as to the mechanism the question has always arisen: Can it not be executed in less time by some other contrivance? Thus every advance has but raised up a new object of rivalry, itself to be superseded by some more rapid means, nor can I hope that I have nearly reached the limit. If I have approached it, it is more than I have a right to expect as the pioneer in this difficult career. Another age must be the judge of that as well as of the other questions relating to the engine. I will only observe that the rapidity of its performance must be judged of first by the velocity which the general structure of the machine enables us to employ with safety. This may in the present engine be greater or less than that I have assigned to it. Secondly, by the number of parts in the time of each cycle which are requisite beyond that necessary for the execution of addition and carriage. Nine units are absolutely necessary for addition and one for carriage and I believe that it may be stated that no engine in which all the digits are added at once can possibly require less than ten units of time for a complete addition or subtraction. Besides this each cycle or turn of the handle must have certain additional units of time allowed it in which the wheels when brought to their right position are locked or rendered immovable and in which also certain motions are given to the axes by which these operations are executed. In the present engine when addition alone is to be executed these extra units amount to five, and when carriage also is required they are ten in number. Thus one of the cycles in which any number can be transferred from one set of figure wheels to another or in which it can be stepped up or down contains fifteen units of time. The other cycle in which addition or subtraction and consequently carriage must be made amounts to twenty units of time.

But it is not merely important that the elements of the operations should be performed in as short a time as possible. The engine ought to be so contrived that in their combination for executing the great operations of multiplication and division the result obtained should be as quickly reached as possible. Several modes have been examined for multiplication and that described has been finally adopted as the shortest.

The reader who only looks at the mechanical difficulties in contriving such an engine will doubtless be surprised when he examines the history of the contrivances, which will be described in a future work, at the lavish rejection of inventions which has taken place in order to achieve rapidity in computation. But the analyst who is aware that the last resource in all our difficulties is the conversion of the most intractable expressions into infinite series will more readily appreciate the importance of even a small abbreviation of the time of an operation, when he is informed that it is one of the objects of this engine to arrive at the numerical value of the coefficients of such series, however complicated the laws by which they are formed, and that proceeding again from these results as etablished data, it is proposed to assign the values of such expressions for any magnitudes of the variables.

In the progress of the invention, it has frequently occurred that mechanical means have either been found altogether insufficient to shorten the time of certain parts of operations, or that it was only possible to effect this object by such great additions of mechanism as were themselves inexpedient.

In this difficulty, I have sometimes had recourse to an arithmetical, or to an algebraical artifice, by which in several instances it has been surmounted without any increase of mechanism. It has sometimes occurred that after the mechanism has been contrived and even drawn, some well-known property of number has enabled me to supersede it. This happened with respect to the means of printing any table *true to the last figure.*

It is well known that though the logarithms of

$$\left.\begin{matrix}2\\3\\4\end{matrix}\right\} \text{ are } \left\{\begin{matrix}.3010299954\\.4771212547\\.6020599913\end{matrix}\right.$$

yet if a table to only seven places of figures is to be printed, the logarithms ought in such a table to be printed:

log of 2 is .3010300
3 .4771213
4 .6020600

In the drawings of the first machine which I contrived for calculating tables by differences there were two arms, one of which after the logarithm had been computed went round to feel if its eighth digit were greater or less than 5. In the former case the arm caused unity to be added to the seventh figure but in the latter case it was inoperative.

After the printing of the seventh figure another arm went in an opposite direction, and subtracted the unity thus added, so that the calculation of the next logarithm might be carried on without impediment from this temporary change. I did not observe until after several contrivances for this purpose had been drawn, that the simple addition of 5 to the eighth figure of the first logarithm computed by the engine would, without any mechanism, ensure the correctness of the seventh figure throughout the whole of the succeeding logarithms of the table.

Another instance of the assistance to be derived from arithmetic occurred in the operation of multiplication. It was found that if the table itself was stepped on its own axes then every alternate turn of the actual multiplication might be saved. The mechanical difficulty which opposed the execution of this plan arose from the necessity of placing those numbers which the digits of the multiplier successively expressed upon the figure wheels of the selecting apparatus. For as this number might be a nine, it was necessary to allow the whole time of one addition, and the movements consequent upon it, or a cycle of fifteen in order to place the number in its proper place; and after this an equal time must be allowed in order to reduce the same wheel to zero, as a preparation for the next multiple. Under these circumstances a remedy without increasing the standard velocity seemed beyond the reach of mechanism.

The following arithmetical artifice removed the obstacle. Let us suppose the multiplier to be the number 584269.[a] If this number be placed on a stepping axis, and stepped down, the lowest figure 9 will be stepped off and lost, but if, instead of this, it be received upon the figure wheel of the selecting apparatus, then the first multiple would be rightly placed. If now a second stepping took place, without obliterating the 9 placed on the selecting apparatus, the number 2 or the second digit of the counting apparatus would be added to the 9, and the result would be a 1, which is not the multiple required.

[a] The number 584269 is sometimes written as 584629 in the text and notations below; since the general argument is unaffected, corrections have not been made.

If this process were continued we should have after each stepping the following corresponding number in the selecting apparatus:

After	1st	2nd	3rd	4th	5th	6th
	5					
	8	5				
	4	8	5			
	6 on selecting wheel	4 on selecting wheel	8 on selecting wheel	5 on selecting wheel	on selecting wheel	
	2 9	6 1	4 7	8 1	5 9	4

So that if we do not obliterate the number at each turn, after putting it in the selecting apparatus, the multiplier 584629 will have the same effect as if it had been the multiplier 491719. This led me to enquire what number must be substituted for any given factor, in order that when so stepped, it may produce successively the digits of the given factor.

The question reduced itself to this: *e d c b a* being the digits of a given factor, what must *E D C B A*, the digits of a number to be found be, in order, that by stepping that number on itself, we may have successively the digits of *e d c b a* of the given factor.

The answer to this question gives the equations:

$$A = a$$
$$B = b - a$$
$$C = c - b$$
$$D = d - c$$
$$E = e - d$$

and in the particular case:

$$A = 9,\ B = 7,\ C = 6,\ D = 2,\ E = 4,\ F = 7$$

and if we place the number 742679 on the stepping axis, the successive results will be, after

1st	2nd	3rd	4th	5th	6th
7					
4	7				
2	4	7			
6 on selecting wheel	2 on selecting wheel	4 on selecting wheel	7 on selecting wheel	on selecting wheel	on selecting wheel
7 9	6 6	2 2	4 4	7 8	0 5

Now the same equations inform us how to form the number thus required. For if we place the given multiplier upon a set of figure wheels, and subtract from it, without attending to the carriage of tens, the same number once stepped up, we shall have precisely those figures which are required, thus:

Factor	5 8 4 2 6 9
Factor stepped up	5 8 4 2 6 9
Subtraction without carriage	5 7 4 2 6 7 9

which number, being used instead of the given factor, will produce its successive digits in the figure wheel of the selecting apparatus.

The same artifice has been employed in another part of the engine, for the purpose of overcoming a similar difficulty in the motion of the circular plate carrying the steel types for punching the copperplates.

With respect to the time employed in the operation of multiplication; if p equal the number of digits in P the smallest factor, d equal to the number of decimals in the engine, and if n is equal to the next whole number which is greater than $P/10$; then the time of multiplication without stepping the product is

$(12 + n)$ cycles of $20 + (p + 4)$ cycles of 15.

But if the product is to be stepped to its proper place previously to its being carried into the store, then the time of the operation is

$(12 + n)$ cycles of $20 + (p + d + 4)$ cycles of 15.

These give respectively in turns of the handle

$$\frac{20(12 + n) + 15(p + 4)}{382}$$

for multiplication without stepping, and

$$\frac{20(12 + n) + 15(p + d + 4)}{382}$$

for multiplication with stepping.

6. *Division*

Of the numerous difficulties which the contrivance of an engine to perform arithmetical operations presented, none certainly offered more reasonable grounds of difficulty than that structure of it, by which it should be enabled to perform the operation of division. Contrary to the plan which might perhaps have been expected, I did not commence the attempt with the case of some simple number as 2, 3, or 5 for the divisor, but went at once to the case of large numbers both for divisor and dividend.

The first rough outline of a system by which it could be accomplished offered an instance of the most complicated mechanism I had ever imagined. But before it was submitted to that simplifying process which invariably succeeds the original conception, another principle occurred apparently so simple in its means, that little could be expected beyond it as to principle, and my whole efforts then became directed to the combinations of mechanism, a great part of which already existed in the engine for other purposes.

It has already been stated, that when any number which happens to be greater than another is subtracted from that other number, an event which has been termed a *running up* takes place, and that in consequence of this event, any order which may be desired can be given to any part of the engine. Bearing this fact in mind, the division of one number by another may be conceived to take place in the following manner:

Let the number 00432101234 be divided by 2018.

In turn[a]	Register	
1. Barrel orders divisor to be	00432101234	
subtracted from highest figure	00201800000	
of dividend and to register	00230301234 + 1	
that it has subtracted once		
2. ” ”	00201800000	
	00028501234 + 1	
3. ” ”	00201800000	
	99826701234 + 1	A running up
4. Addition of divisor	00201800000	happens which
Unregister one	00028501234 − 1	directs the addition
5. Step up dividend	0028501234	of the divisor and
6. Barrel orders divisor to be	0020180000	then the stepping
subtracted as before	0008321234 + 1	up of dividend
7. ” ”	0020180000	
	9988141234 + 1	A second running
8. Addition of divisor	0020180000	up and the same
Unregister one	0008321234 − 1	orders as above
9. Step up dividend	008321234	

[a] Several corrections have been made to the diagram. An arithmetic error in turn 26, which should give the result 0050560 instead of 0049560, has not been corrected.

10. Subtract divisor	002018000	
	006303234 + 1	
11. ”	002018000	
	004285234 + 1	
12. ”	002018000	
	002267234 + 1	
13. ”	002018000	
	000249234 + 1	
14. ”	002018000	
	998231234 + 1	Running up
15. Add divisor	002018000	and the same
	000249234 − 1	orders as above
16. Step up dividend	00249234	
17. Subtract divisor	00201800	
	00047434 + 1	
18. ”	00201800	
	99845634 + 1	
19. Add divisor	00201800	
	00047434 − 1	
20. Step up dividend	0047434	
21. Subtract divisor	0020180	
	0027254 + 1	
22. ”	0020180	
	0007074 + 1	
23. ”	0020180	
	9986894 + 1	Running up
24. Add divisor	0020180	and the same
	0007074 − 1	orders as above
25. Step up dividend	0070740	
26. Subtract divisor	0020180	
	0049560 + 1	
27. ”	0020180	
	0029380 + 1	
28. ”	0020180	
	0009200 + 1	
29. ”	0020180	
	9989020 + 1	
30. Add divisor	0020180	
	0009200 − 1	
etc.	009200	

In the above view the first direction given by the barrels is to subtract the divisor from the dividend. This order is repeated until at the end of the third subtraction, it is found that a *running up* has taken place, a fact which of course indicates that the divisor has been subtracted once too often. The number of times the divisor has been subtracted is marked by a register axis. Now it is in consequence of this fact of a running up that the engine *knows* it has subtracted the divisor exactly once too often, and that knowledge is conveyed by the running up lever to the barrels, in the form of an order to move to such verticals, that at the succeeding turns they shall direct the erroneous subtraction and registration to be corrected, and the remainder of the dividend to be stepped up in order to undergo a similar process for the discovery of the next digit in the quotient.

It may perhaps be observed that in the above example another *running up* occurs in the fourth turn at the ninth line in consequence of addition, and it might be expected that an order should consequently follow. But this is provided against by the very general nature of the running up levers, which are capable, under different circumstances, of conveying any order that may be required to any part of the engine. Immediately upon the occurrence of a running up these levers, besides the directions already stated, convey through the barrels an order to themselves to be inefficient at the next running up, thus the runnings up in the 8th, 15th, 19th, 24th, etc., turns are inefficient, whilst in the 3rd, 7th, 14th, 18th, 23rd, and 29th, they are efficient.

The first figure in the quotient being thus obtained, a repetition of the same process will of course produce the succeeding figures.

The complete understanding of this principle is of some importance, for it not merely relates to the process of division, but it is capable of being applied to any tentative arithmetical process. I had applied it to the extraction of roots, but other improvements induced me to dispense with it in that instance. Its application is not limited to existing rules, but I believe I may venture to state, that whenever new methods are contrived for overcoming arithmetical difficulties by tentative processes, it will be found available for those new purposes, and that even in cases where the tentative processes relate to algebraic expressions, it may yet be useful.

I have given a simple case in order to explain the nature of this principle. Division however is not executed by this tedious process, which would require for its performance a number of turns more than equal to the sum of the digits in the quotient. The process really used may be thus described:

The operation card ordering division, having advanced, directs the barrels to the proper verticals.

The divisor P is first received from the ingress wheel upon several parts of the mill.

The dividend R next enters. Several processes are gone through in order to ascertain whether it will be necessary to step the dividend up or down, and how many steppings there ought to be of either kind. This depends upon the number of digits of the quantities operated upon, and is principally executed by the counting apparatus which also computes the number of figures which must occur in the quotient.

At about the eighth turn the formation of the table commences, and also the selecting apparatus is prepared.

If the stepping down is completed, the actual division may commence about the eighteenth turn.

By means of the selecting apparatus the two first figures of the dividend are compared with the two first figures of each of the nine multiples of the divisor, and that multiple which has its two first figures either nearest in magnitude or equal to the two first figures of the dividend is selected and subtracted from the dividend.

The remainder is then stepped up, and the same process repeated until the counting apparatus interferes to finish the process.

In case the two first figures of the dividend are exactly the same as the two first figures of one of the multiples of the divisor, there may arise a doubt whether that multiple or the preceding one should be subtracted; this will depend on the third figure of each quantity. But in this case, the mill always takes for granted that the multiple selected is the true one. If, however, the contrary happens to be the case, then the subtraction of the multiple which is really too large will cause a running up on the dividend, and in consequence of this, orders will be transmitted to the barrels to direct the addition of the first multiple, and the subtraction of unity from the multiple registered in the quotient, by which means the quotient being set right, the first process is then continued.

After the end of the actual division the quotient, and the remainder if required, are given off to the egress wheels.

As the result of a multiplication of a number containing forty digits by another of equal extent is a product containing eighty digits, and as this product can be conveyed into the store on two sets of figure wheels, a provision has been made for bringing such numbers back into the mill when they are to become dividends, so that the result of the division of a number containing eighty by one of forty digits, shall have the forty figures of its quotient true to the last figure.

In deciding on the number of the first figures of the dividend to be compared with those on the table, it was considered that the infrequency of the running up when two only are used, was such that the resulting loss of time would be more than counterbalanced by the mechanism required in case three figures should be compared. If however it should ever be thought expedient to save that portion of time, it is quite possible by adapting a carrying axis to each table axis to render division a direct process, in which no figure can ever be assumed in the quotient which requires a correction afterwards.

The process of division occupies more time than that of multiplication, a circumstance partly arising from the greater extent of the preliminary process, but chiefly from the necessity of occupying one turn of the hand for stepping down, which is executed in multiplication during the time of the addition.

Whenever therefore by proper arrangement of a formula or by other artifices multiplication can be substituted for division time will be gained by the change. Examples of this exchange will be found frequently in the following pages.

The time occupied by the process of division will depend on the numbers concerned, and on the position of the decimal point, and in a small degree on the accidental number of the runnings up.

. . .

7. *Extraction of the square root*

I had at one period in the progress of the engine determined on making it perform the operation of extracting the square root of numbers. For this purpose the usual method was in some degree modified and adapted to the structure of the mill. Advantage was taken of the effects of running up to alter the various directions necessary for the process, and notations were made for the purpose. The time occupied in obtaining each figure was about four turns for every unit it contained, and the process therefore rather long. When successive improvements had considerably diminished the time of the action of the engine, I gave up the process at first intended for the extraction of the square root, and the grounds of the change were:

That with the shortened cycle the process of obtaining the square root by successive approximations would not be longer than the direct extraction in the former state of the engine.

That considerable simplification occurred in the machinery from giving it up.

That the extraction of the cube and all other roots would then be performed according to the same principles.

In the engine now described the extraction of square and other roots will be performed by approximation. If the quantity whose square root is required has a negative sign, the arithmetical root of the number will be found, but a particular notice will be given by which the circumstance that the root so found must be multiplied by $\sqrt{-1}$, will also be indicated. The mode of executing these approximations will be explained in a future page.

IV OF COMPUTING THE NUMERICAL VALUE OF ALGEBRAIC FORMULAE

It will have been observed that the orders by which the engine has been directed to execute the elementary arithmetical operations already described have been of two kinds: operation cards having certain holes by which they govern the barrels, and the variable cards.

These operation cards are always identical with each other, as long as the same operation is to be performed, and they are totally independent on the particular numbers which they may direct to become subject to them. In the cases of multiplication and division, the operation cards will indeed be turned and advanced at the requisition of the barrels in a longer or shorter time according to the nature of the numbers, but the *forms* of these cards will always remain unaltered. In this sense the operation cards partake of that generality which belongs to the algebraic signs they represent.

For the processes of addition and subtraction two operation cards only are necessary. One of these is required for each quantity, and assigns to it the algebraic sign of the process. Thus any number of additions and subtractions are ordered by the same number of operation cards.

Besides these two operation cards it is necessary to have three variable cards. The two first direct the two variables to be operated upon to move from the store into the mill, the third variable card directs some particular store axis to receive the computed sum of the two quantities.

Whatever be the number of quantities connected by the signs plus or minus, the same number of operation cards is required, and the same number increased by one expresses that of the variable cards.

In the processes of multiplication and division, one operation card is required to order the barrels to commence the process. If it should be necessary, under any circumstances, to have two different modes of performing these operations as is at present proposed, for the purpose of saving time in approximations, then another card may be attached, which is to be a blank card, and therefore ineffective in the most frequent case, but which is capable of bringing in the less usual case when it is required.

Besides these operation cards, two variable cards will be required to order in the two factors, and three others to order the three parts of the product into their respective places on the store.

In division three variable cards will be required to order the dividend out of the store into the mill, and one more to direct the quotient to its right place in the store.

The following table exhibits the number of cards necessary for each operation.

	Operation cards	*Variable cards*
To add two quantities	2	3
To subtract one quantity from another	2	3
To multiply two quantities and retain 80 figures of product	1	5
To divide one quantity of 80 figures by another quantity	1	5

In any operation it is only necessary that the operation card ordering its commencement should be advanced. This sets the barrels in action and provides, through the variable cards if necessary, the first quantity to be operated upon; the barrels then call in the other quantities required at the proper times, until the operation is completed, the mill cleared of all numbers, and the computed result placed in its intended situation in the store. The last vertical on the barrels belonging to each operation directs them to move to its zero point.

Such being the arrangement, the calculation of the numerical quantities resulting from two or more successive operations may be readily performed by placing the operation cards strung together in their proper order on their revolving prism, and the variable cards marking the numbers in the store, on which are to be the subject of

those operations, and also in the order in which they are required, on their own revolving prism, and then putting on the *final verticals* of one of the barrels a stud which orders the operation cards to advance.

This arrangement being made, after the first advance of the operation cards, the barrels will direct the progress of the operation, calling into the mill by the advance of the variable cards the numbers to be operated upon, and replacing in the store the computed result by the same means. The barrel, by the return to the final verticals of this operation, orders another operation card to advance, which, giving a new order to the barrels, these latter again conduct that operation, demanding at proper times the fit numbers from the store, and conveying back to it their new result, and finally, by returning to their final verticals, they again demand a fresh order from the operation cards.

1. *Verification of the numbers placed in the store*

The numerical quantities intended to be operated upon may be placed by hand on a particular set of figure wheels in the store. Or they may be punched previously by a separate machine on number cards, and thus conveyed into the store.

In either case the possibility of error exists, and although it might reasonably be required that no care should be spared in inserting in the engine correct data for the question whose solution is demanded, yet it is important that the fulfilment of this condition should not depend *solely* on the superintendent of the engine, but that any mistake he might make should be open to detection when the calculated results are returned to those who supplied the formula and constants with which they were to be computed.

Fortunately this condition, which really becomes of great importance when the number of the given constants amount to hundreds or thousands, can be easily fulfilled.

It is merely necessary to order by means of cards that each constant, as it enters the engine, shall pass through the printing apparatus previously to its taking its intended place in the store. By this means the numbers really put into the engine by the attendants will be printed and they may be returned with the computed results printed also by the engine upon the same paper. If the calculation consist of the reduction of a multitude of observations it may be convenient to print, in an adjacent column, each observation entered into the engine and its corrected result.

Thus whatever may be the formula ordered to be computed, the employer of the engine will always possess ample means of detecting any error in the constants inserted in it by the attendant.

2. *Verification of the formulae placed on the cards*

The methods which have been contrived for accomplishing this object are not so simple as that for insuring the accuracy of the constants, nor are they, without the use of considerable machinery, so absolutely certain in their results.

It must, however, be observed, that if care is demanded from the attendants for the insertion of numbers which are changed at every new calculation of a formula, any neglect would be absolutely unpardonable in combining the proper cards in proper order, for the much more important purpose of constructing the formula itself, the arrangement of whose cards is never changed at any after time. This verification might therefore be reasonably left to the diligence of the superintendent.

There are however several subsidiary modes by which he may himself check his own proceedings.

The operation cards, for example, may be made of four different colours corresponding to the four operations of arithmetic:

a white card indicating addition
a yellow card indicating subtraction
a blue card indicating multiplication
a green card indicating division.

When therefore the chain of operation cards is finished, their total number ought to be equal to that of the operations in the formula. The number of white cards ought to agree with the number of additions, the number of yellow cards with that of the subtractions, whilst the number of blue cards marks the number of multiplications, and that of green cards the number of divisions.

Since the number of quantities which enter the mill to be operated upon, as well as that of the results to be returned to the store, is known, the number of the variable cards is known. These might be coloured like the operation cards, and those cards which ordered numbers out of the mill into the store might be coloured balck. If this were the case, before any two sets of formula cards were used they might be compared as to the succession of their colours, when, if correctly placed, it would be found that the same succession of colours would occur in each, but that

in the variable cards each colour would be separated from the preceding by one or more black cards.

Another mode of partially verifying a formula consists in assigning such numerical values to the constant quantities as shall render its value easily computed by the pen. If either the operation or the variable cards have been wrongly put together, the formula given to the engine will be different from that which was intended, and the numerical results will in all probability be totally different from that computed by the pen. If trials of three or four simple cases have been made, and are found to agree with the results given by the engine, it is scarcely possible that there can be any error amongst the cards.

When the formula to be computed is very complicated, it may be algebraically arranged for computation in two or more totally distinct ways, and two or more sets of cards may be made. If the same constants are now employed with each set, and if under these circumstances the results agree, we may then be quite secure of the accuracy of them all.

It should, however, never be forgotten, that whatever attention may have been bestowed on securing the accuracy of the cards, that object, when once attained can never again become doubtful.

If, notwithstanding these methods of verification, it should still be considered desirable to have greater security, it is certainly possible to attain it by making the engine itself print the formula inserted in it, as well as the constants to be employed in that formula.

At two different stages in the progress of the invention, I had contrived mechanism by which this might be accomplished, but as it related to a much more advanced stage, I did not make any drawings of it. Since the introduction of formula cards, these contrivances have become susceptible of great improvement. For it is now no longer necessary to add mechanism for that purpose to an engine already sufficiently complex; but a separate machine should be constructed, by placing the formula cards in which the formula might itself be printed from them.

Such a machine does not appear to me to present much difficulty, as far as regards selecting the signs and variables. The chief inconvenience would arise from the uncertain extent of the compound quantities operated upon. This might in a great measure be obviated by only requiring it to print the series of successive substitutions which are to be made in the engine. The reader will understand the force of these remarks better when he has perused the mathematical part of this volume.

But even this assistance might fail in some of the more complicated cases, for a certain degree of doubt always remains whether we have ourselves worked out rightly the developed results which we wish to convert into number. This doubt may be almost altogether removed when we avail ourselves of the *combinatorial cards*, because in such instances, although the ultimate results are extremely complicated, yet the instructions communicated to the engine by cards are very few, and exceedingly simple, and may therefore be repeatedly verified in a very short time. It appears however to me that the formula printing-machine might by some improvements itself ultimately work out many of such algebraic developments.

3. *Extent of the numerical power of the engine*

In deciding on the extent of the numbers with which the engine should compute, the first consideration was to look at the number of figures which in the present state of mathematical enquiry are required in the most extensive calculations. It then became desirable to look forward to the probable increase which improved observations might require. Finally, the mechanical structure of the engine, or its necessary arrangement, might put limits to this extent, or render the time employed in given calculations longer, at certain definite intervals.

But even if it were thus possible to satisfy all practical wants there would still remain a desideratum to render the mechanism philosophically perfect as to its power of converting algebraic expressions into numbers, without any limit as to their magnitude or extent.

The result of my reflections has been that numbers containing more than thirty places of figures will not be required for a long time to come. I have, however, made the drawings of the engine for forty places of figures. All additions and subtractions may be made with such numbers and the products, amounting to eighty places, may be preserved in the store and brought back into the mill to be divided by numbers of forty places of figures, thus retaining forty places in the quotient.

I have however considered thirty places of figures as the standard, and upon that basis have made all the estimates of the time required for the computation of formulae. In order to fulfil the condition of the conversion of algebraic formulae into number without reference to the number of digits in the quantities employed, I have availed myself of certain arithmetical principles, by the aid of which, without adding to

the mechanism, calculations, if requiring any extent of digits, may be performed; these principles will be explained in a future page. At present it may be sufficient to observe that if it is required to compute formulae having in each quantity not more than eighty figures, it will be sufficient to employ about four times the time requisite for the calculation of the same formulae, having only half that number of digits in each quantity. If three times the number of digits (or 120 digits) be required in each number, then about nine times the time will be requisite and in general, if the number of figures required to be used in the given quantities be expressed as n times the number forty, then the time required by the engine will be: $40 \times n^2$.

It was important, at least for the theoretical perfection of the engine, that no additional mechanism should be required for any extent of numerical calculation, beyond that which was considered to be safely practicable in the present state of the mechanical arts.

This point has been accomplished with respect to numbers, by increasing the time nearly in proportion to the square of the number of times the sum of the digits is a multiple of forty.

As to the extent in point of successions of the algebraical operations, no practical limit seems to exist. The number of cards which indicate the operations, and those that govern the quantities, may be increased to any extent without loading the engine, and in the more complicated developments the combinatorial cards may be made use of, and expansions, which by the usual means would require above twenty thousand cards, may be performed with little more than two hundred.

4. *Time required in computing formulae*

The most constant difficulty in contriving the engine has arisen from the desire to reduce the time in which the calculations were executed to the shortest which is possible.

It is not to be presumed that such an attempt has succeeded. How near the approach has been made must remain for aftertimes to determine. It is, however, desirable to state the mode in which the time has been estimated and to point out sources of mistake which might perhaps be overlooked by those who have not deeply studied the subject.

The first step is to assume as a definition, the unit of space and the unit of time. I have assumed for the first of these, the space (measured on a circle), between the figures on the figure wheel. This is ____. The unit of time is assumed to be that portion of a second required for the passage of one figure to another on the same wheels.

The time occupied by one turn of the handle of the engine, which is called a cycle, consists of the time necessary for the directive action, the time requisite for executing those directions, the time necessary for addition, the time necessary for locking the wheels moved, the time necessary for carriage, and the time required for locking the wheels after carriage.

It sometimes occurs that carriage is not required, in this case the cycle consists of fifteen units of time. When carriage also is to be used, twenty units of time are necessary.

The first mode of reducing the time which presents itself is by diminishing the unit of space, or the distance between the figures. The first inconvenience which presents itself is that the machinery becomes more delicate and requires better workmanship. The general scale on which it is constructed becomes that of clockwork, and finally is reduced to watchwork. This evil will of course be diminished as the mechanical arts advance in perfection.

Another consequence of the diminution of the unit of space is, that as the *time lost* at each gearing is not proportionally diminished, it becomes necessary to make more frequent corrections by means of additional locking apparatus. The time required for these lockings increases the length of the cycle . . . the unit of space was assumed to be ·62832 and a correction was applied to destroy the loss of time at every third wheel. When the two counteracting principles of a short unit and many lockings were distinctly perceived the present unit was adopted as that most fitted for the present state of mechanical art and a locking was placed at every second wheel.

The time necessary for the action of the directive part consists in that employed in advancing the barrels, and in turning and advancing the several classes of cards. The advance of the barrels and that of the cards can be made contemporaneous, and the extent of their motion will depend on the mode by which the several parts are thrown into gear, and the time will depend upon the space, and the mass of matter to be moved.

The time necessary for the circular motion of the barrels from one vertical to another, and also that required for the turning over several cards to arrive at the one wanted, when it is at some distance from that last used, must also be considered. This operation may be executed

during the addition part of the cycle, provided the distance between the vertical in the barrels and the cards on the prism is not too large. When the unit of space ·62832 was employed, fourteen verticals could be passed over in a cycle and ____ cards. But when that unit was reduced to ____ it was found that only 5 and 7 verticals would be passed and ____ cards in the same portion of the cycle. The consequence of this diminished time will be that under certain circumstances it may become necessary to employ an extra turn of the handle in order to bring the barrels or the cards to their proper positions. This then forms the first counteracting principle to be considered when it is proposed to shorten the unit of space.

The corrections for loss of time arising from the original structure and subsequent wear of the teeth of the wheels are made by means of the locking apparatus. Levers terminating in wedges are forced between the inclined planes of the locking teeth, and thus bring the wheels to which they are attached accurately to their places. The depth of the locking teeth depends on the extent of the correction, and the time occupied in moving them will depend partly on that depth, and partly on the clearance required when they are unlocked. If therefore the same extent of motion is carried through a short and through a long train of wheels, the time occupied in bringing the last wheel to its true place will in the first case arise from few lockings moving through certain spaces, whilst in the long train it will result from more numerous lockings, each of which moves through a smaller space.

If the space required for clearance could with safety be diminished in the same proportion as the depth of the lockings, then in point of time merely, it would be indifferent whether the lockings were few or many, and their number would then be determined by the simplicity of mechanism, and by other principles. The unlocking of the several wheels may take place in a much shorter time than that required for locking them.

The time requisite for executing the orders of the directive part consists in that during which the original source of movement is connected with the several parts which are ordered to be moved. The time of addition comes within this class, and so also does that in which certain gearings are made and broken by the lifting or descent of certain axes. As these gearings can usually be made at the same moment, / their number does not augment the time they employ. But if, in order to save mechanism, those axes move to several points of rest and consequently pass over considerable space, then more time must necessarily be allowed. When the number of figure wheels in each cage was reduced from four to two, the time employed in lifting these axes was also reduced.

The time consumed in carriage varies more than that of any other portion of the cycle, according to the different mechanism employed for that purpose. The mere time of adding the units caused by carriage may occupy one unit of time or it may require forty. The time for the previous adjustment will also vary but in a less degree. It is therefore on this point that by far the most important saving can be effected, a saving which would justify even very expensive machinery.

In the earlier stages of the invention of the engine, the great extent of the mechanism for carriage, which it was necessary to attach to each set of figure wheels, became a serious obstacle to its advancement. It was the pressure of this difficulty which led me to reflect on the great assistance derived in analysis, from separating the symbols of operation from those of quantity, and by a distant analogy to attempt the introduction of a carrying apparatus, which might be attached, as occasion required, to any set of figure wheels.

When I reflected on the fact that the quantity of mechanism necessary for performing the operation of mere addition without carriage was not above the tenth part of that requisite for carriage itself, the importance of such a separation became evident, and this led me to separate the machine into two parts, one (the mill) in which operations were performed the other (the store) in which the quantities used and produced were inserted.

In its present state the time by which the cycle is increased when carriage is required is five units, only one of which, however, is employed in the actual carriage. The rest of the five is filled up by the necessary motions for preparing carriage, and for the additional lockings consequent upon it.

The introduction of the principle of hoarding carriages until they amount to nine, has in the case of multiplication enabled me to reduce that operation still further by substituting *nine short* and *one long* cycle, for every *ten long* cycles previously employed.

The time necessary for locking the wheels after carriage will be nearly the same as that requisite for locking them before. According to the nature of the mechanism a gearing or two more or less will make the only difference.

Previously to deciding on the absolute time of each cycle, another question must be entered upon. Since the most important circumstance

which affects the duration of a cycle by a time amounting, even in the most favourable circumstances yet contrived, to one third of that in which addition can be performed, is the time which is required for carriage, it may reasonably be enquired whether, by adopting a different radix for the arithmetical notation, we may not in some measure avoid the loss of time. I accordingly at one period considered the advantage of adopting the arithmetical notation of which the base is 100 instead of 10.

The immediate consequence of such a plan was that carriage occupied on the average about one ninth of the usual time. Some drawings were made for this view of the subject but after much consideration the plan was given up as undesirable even in an engine of differences.

Having thus examined the elements of the operations, and pointed out the chief ingredients of the time they occupy, the next consideration is their combination into operations. As far as I have been able to examine the processes of addition and subtraction, it does not appear that much is to be gained, since the lowest limit must be one turn for each process and the limit is at present $n + 4$ turns for n operations. The small saving which might possibly be contrived is not of the same importance as any economy in the time of performing the larger operations.

With respect to multiplication two methods may be followed, either the process may be commenced by taking that multiple out of the table which corresponds to the lowest or units figure of the multiplier, and then proceeding to the multiple corresponding to the tens, and so on; or we may begin by selecting the multiple corresponding to the highest figure, then that to the second figure and so on.

The first of these modes of proceeding requires only one carriage, because at each stepping down of the product one figure is carried off to another axis, and no subsequent addition to the product can ever alter the figures so transferred. If, however, we commence at the highest figure of the multiplier two carriages become necessary, because any multiple, down even to that which the units figure directs, may by possibility change all the others up to the highest, and since the product of forty figures by forty produces a number of eighty places of figures, this must be placed upon two sets of figure wheels and consequently require two carriages.

The operation of division does not appear to admit of the same two methods as those which are applicable to subtraction. It appears to be necessary to perform subtraction, and then step up the remainder, and any attempts I made to unite the two processes so as to perform them in one cycle have proved ineffectual.

. . .

The time occupied by the engine in computing the same formulae will be different according to the mode in which it is arranged. This will be apparent from a single case, let

$$x = \frac{a^2c + b^2a + c^2b}{abc}.$$

This will, in its present form, require eight multiplications, two additions, and one division.

If, however, it be put into the form

$$x = \frac{a}{b} + \frac{b}{c} + \frac{c}{a}.$$

it will only require three divisions, and two additions. In the latter case the time of computing it will be but little more than half that required in the former.

The difference is still greater in other cases. If it is required to find the arithmetical value of

$$a^6 + 5a^5b + 10a^4b^2 + 20a^3b^3 + 10a^2b^4 + 5ab^5 + b^6$$

each term will require five multiplications besides that which arises from its numerical coefficient, so that even ommitting the time so occupied, the computation will require thirty-five multiplications and six additions whilst if it be put under the form

$$(a + b)^6$$

the same calculation may be made by five multiplications and one addition.

The consequences resulting from this circumstance are important, and deserve the attention of those who are engaged in extending the domain of analysis, as well as of those who look forward to the effects which are likely to be produced by the complete control which mechanism now gives us over number.

Whenever engines of this kind exist in the capitals and universities of the world, it is obvious that all those enquirers who wish to put their theories to the test of number, will apply their efforts so to shape the anlytical results at which they have arrived, that they shall be

susceptible of calculation by machinery in the shortest possible time, and the whole course of their analysis will be directed towards this object. Those who neglect the indication will find few who will avail themselves of formulae whose computation requires the expense and the error attendant on human aid.

LIST OF PLATES

Thirteen plates of the Analytical Engine[a]

[a] These plates, reproduced from *Babbage's calculating engines* (London, 1889), are not dated. Some of the proofs of the plates in the Babbage papers, Waseda University, are dated as follows: 3 and 4, 10 May, 1837; 8–10, April, 1837; 12, 4 December, 1836. See 'Sketch of the Analytical Engine', this volume, pp. 91–2, for a description of the plates.

[b] Plates 2–7 should be considered as a group. The complete plate is given in Plate 6, and with lettering in Plate 7. Plates 2–5 show various parts of the complete plate.

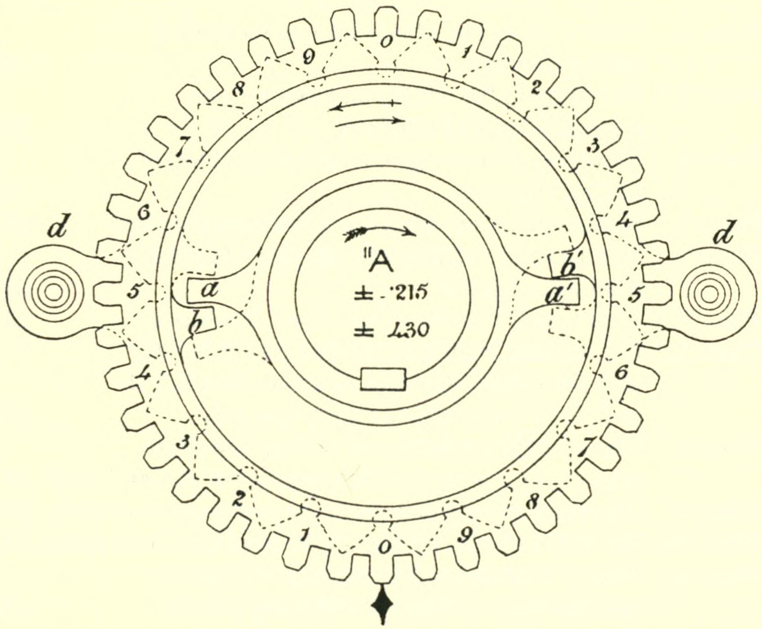

PLATE 1

Plan of the figure wheels for one method of adding numbers

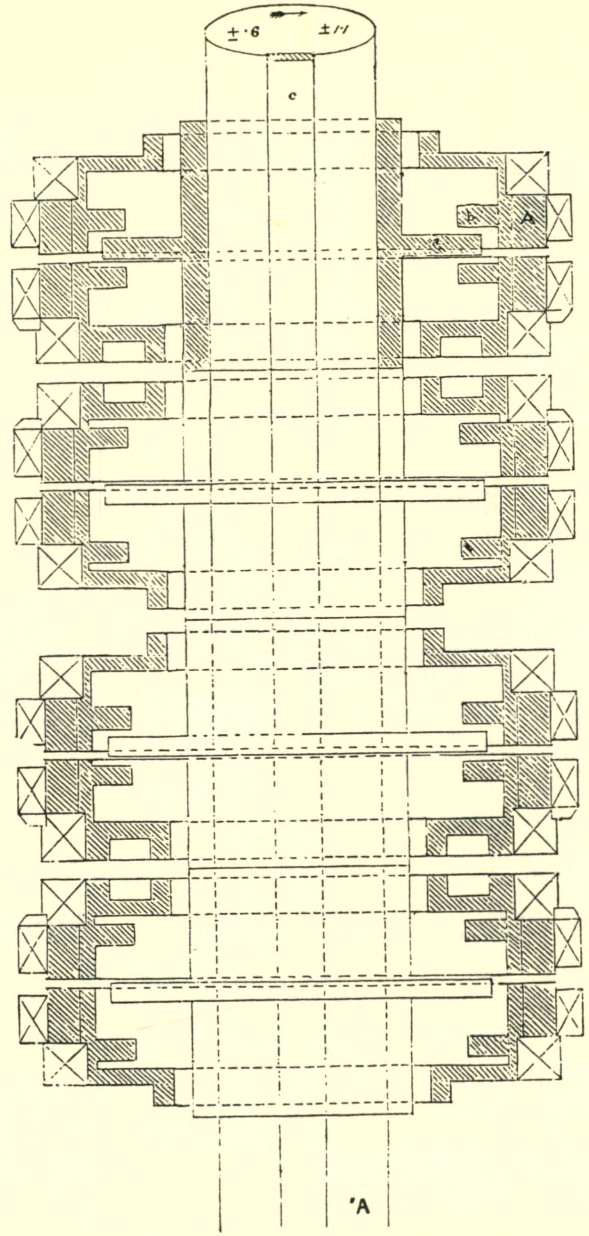

PLATE 2

Elevation of the wheels and axis for one method of adding numbers

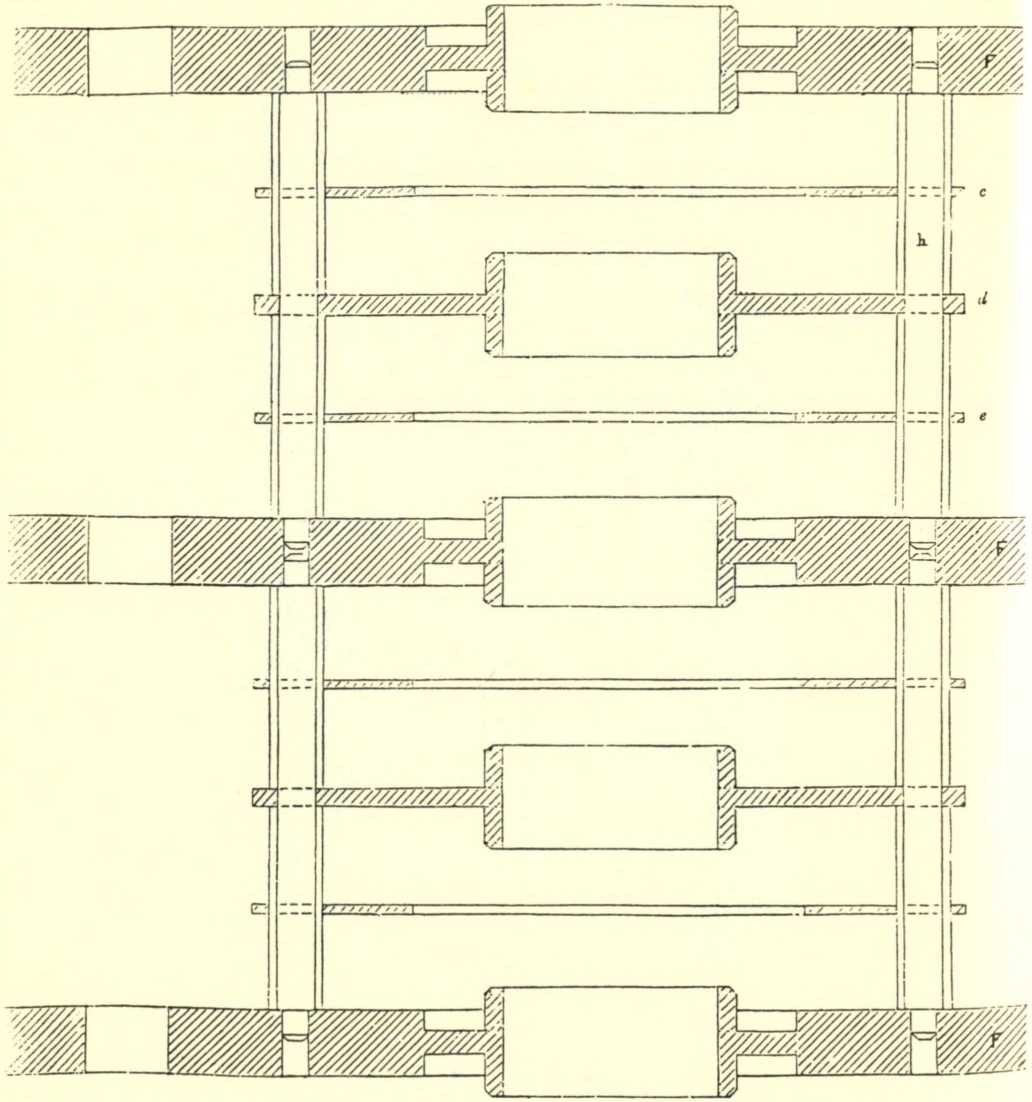

PLATE 3

Elevation of framing only for one method of adding numbers

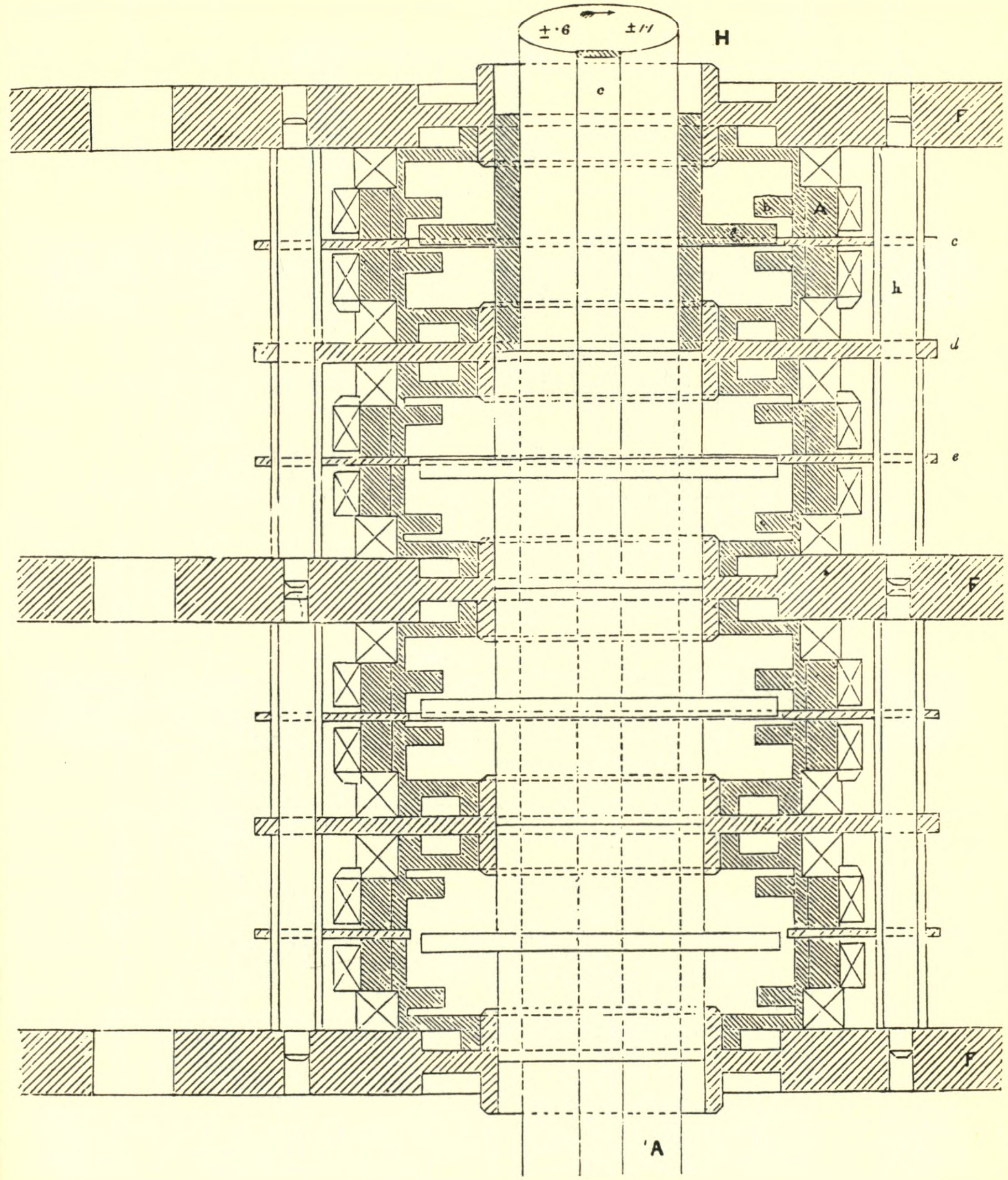

PLATE 4

Section of adding wheels and framing together

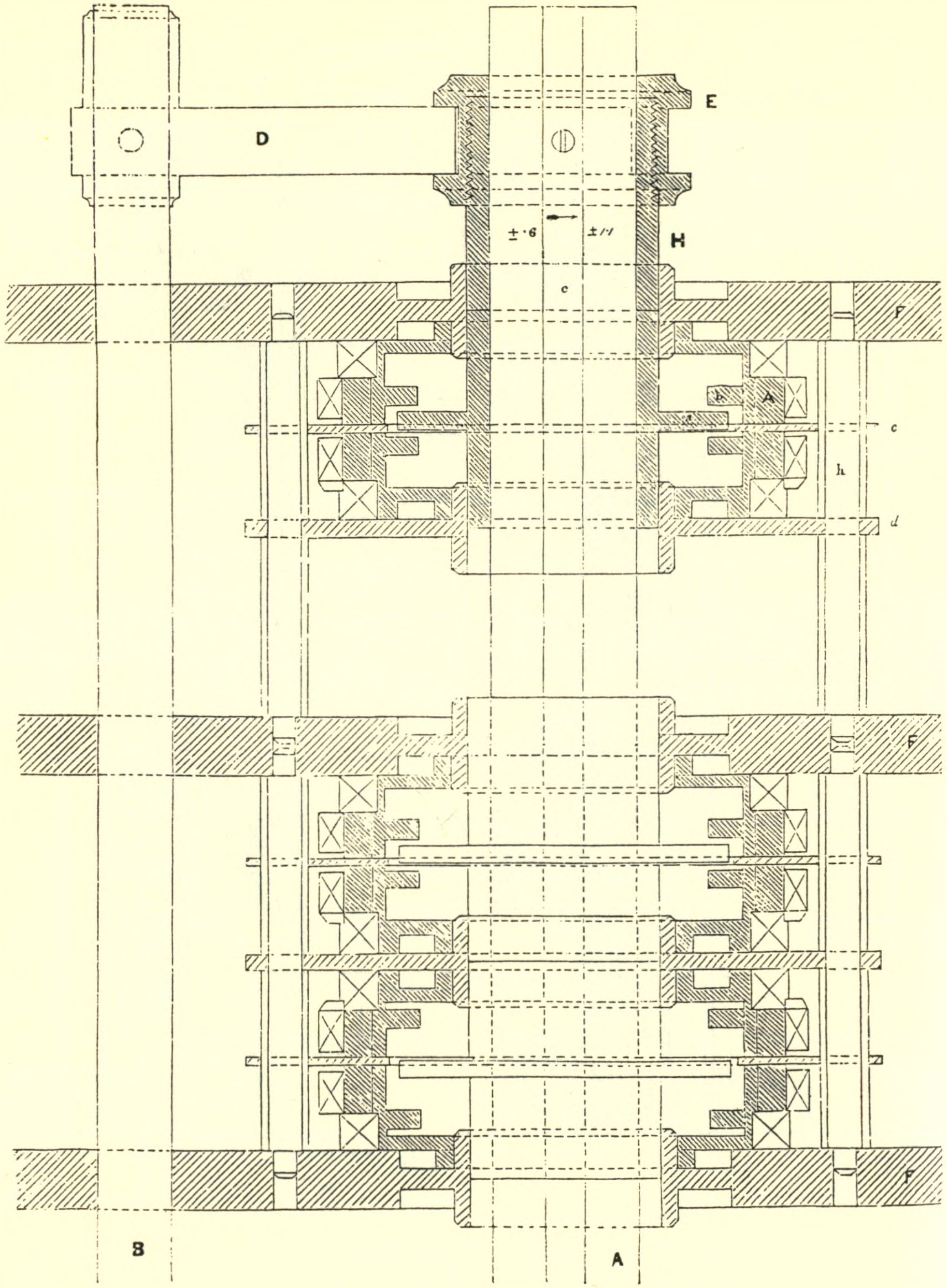

PLATE 5

Section of the adding wheels, sign wheels and framing complete

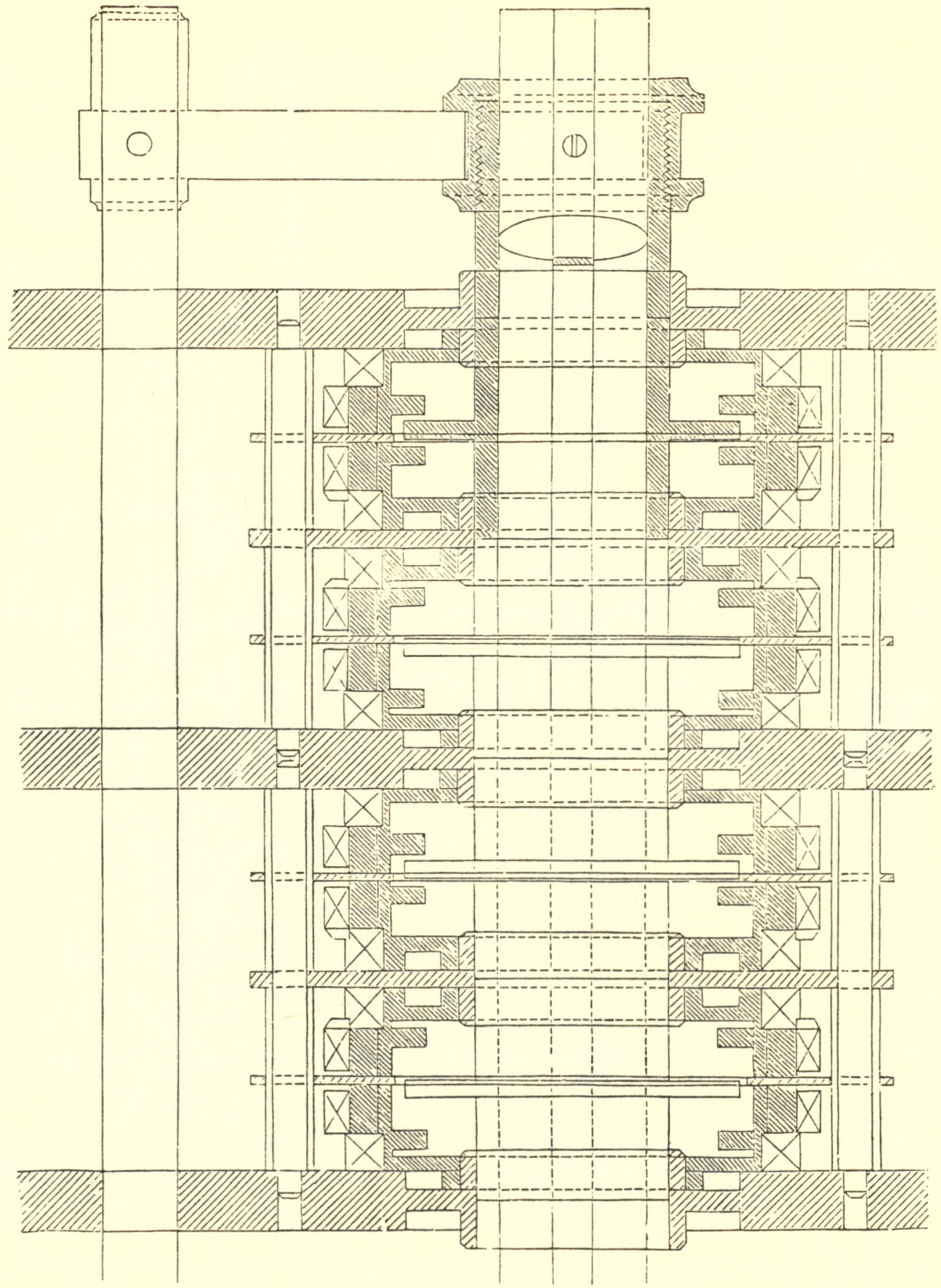

PLATE 6

Impression from the original wooden block

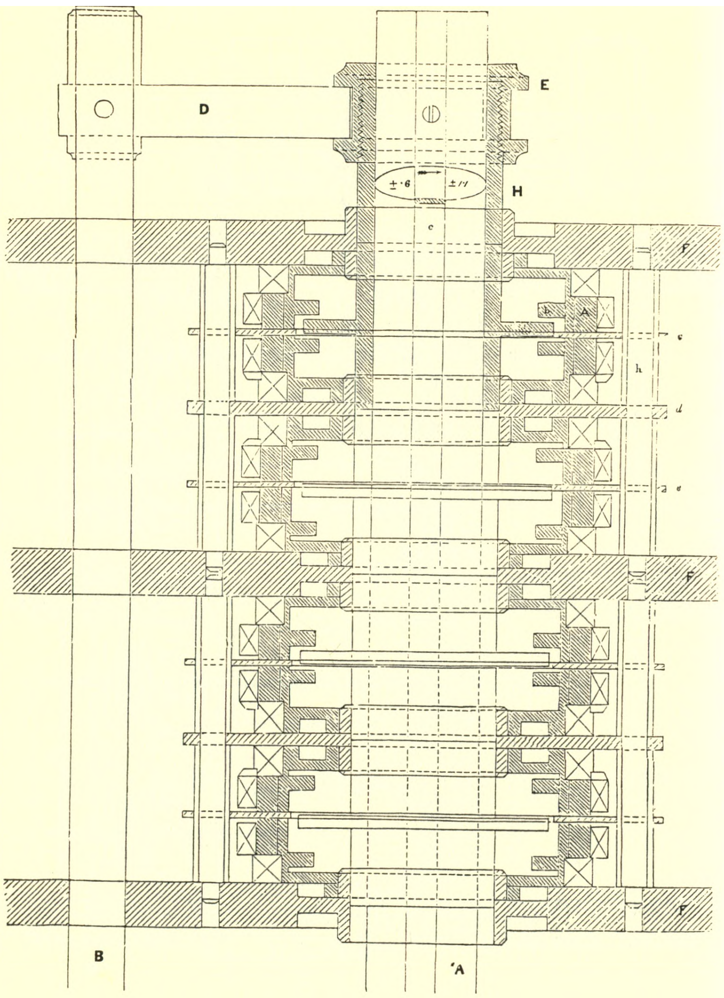

PLATE 7

Impression from a stereotype cast of No. 6, with the letters and signs inserted. Nos 2, 3, 4 and 5 were stereotypes taken from this

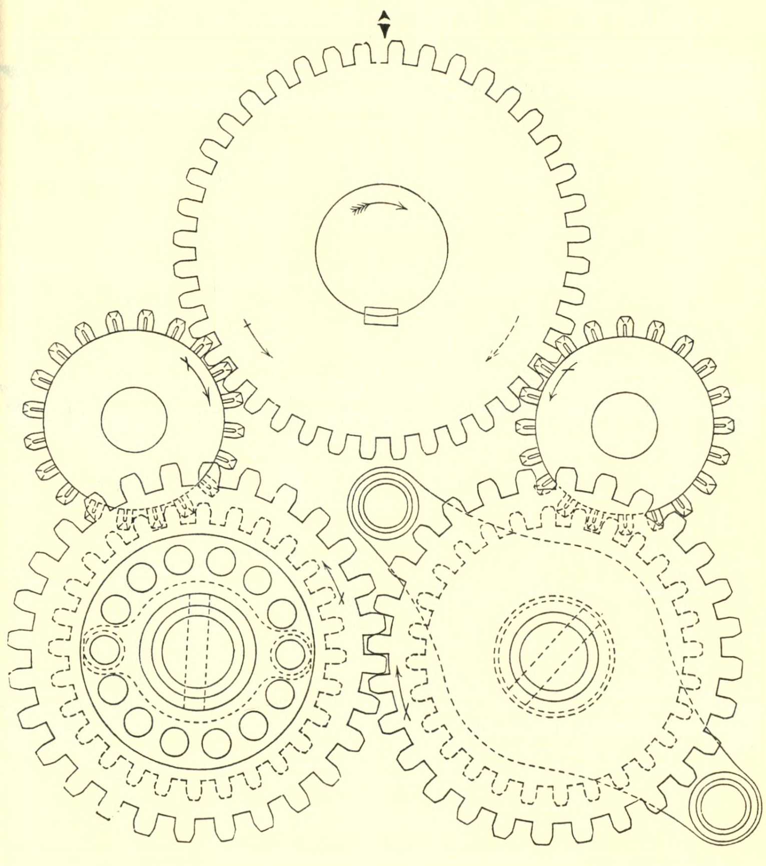

PLATE 8

Plan of adding wheels and of long and short pinions, by means of which *stepping* is accomplished

N.B. This process performs the operation of multiplying or dividing a number by any power of ten

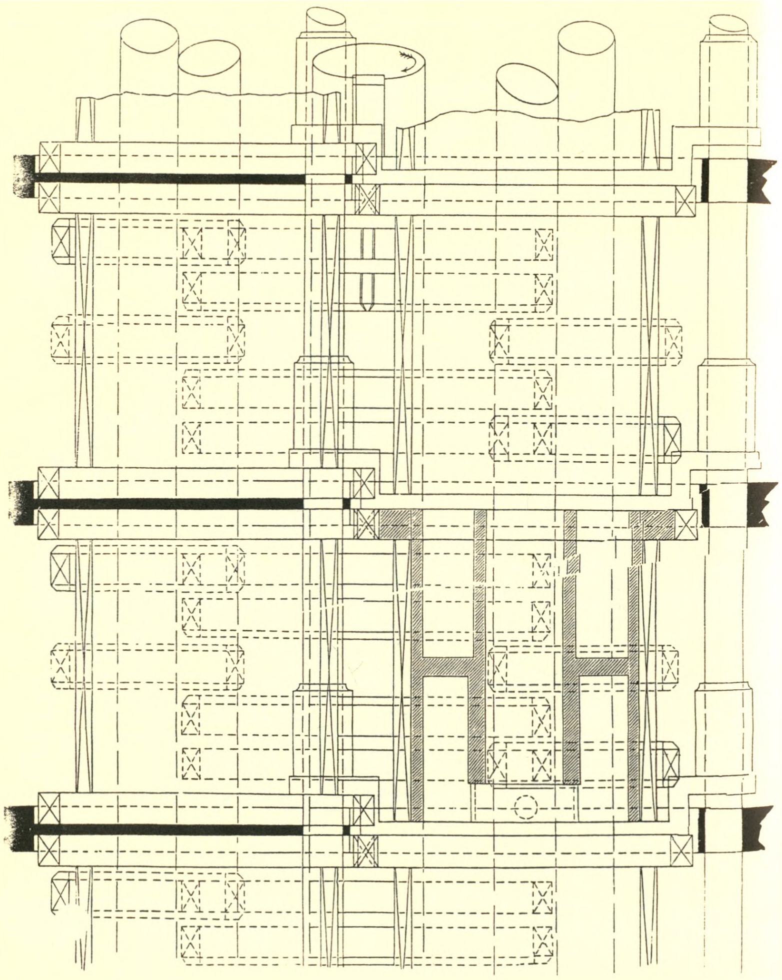

PLATE 9

Elevation of long pinions in the position for addition

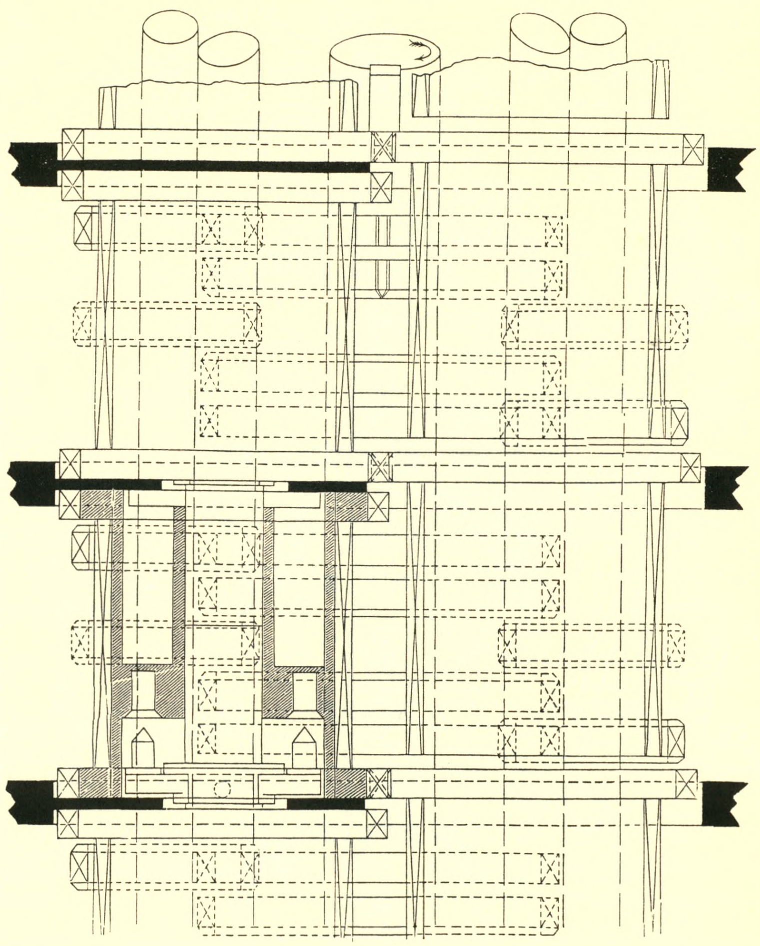

PLATE 10

Elevation of long pinions in the position for stepping

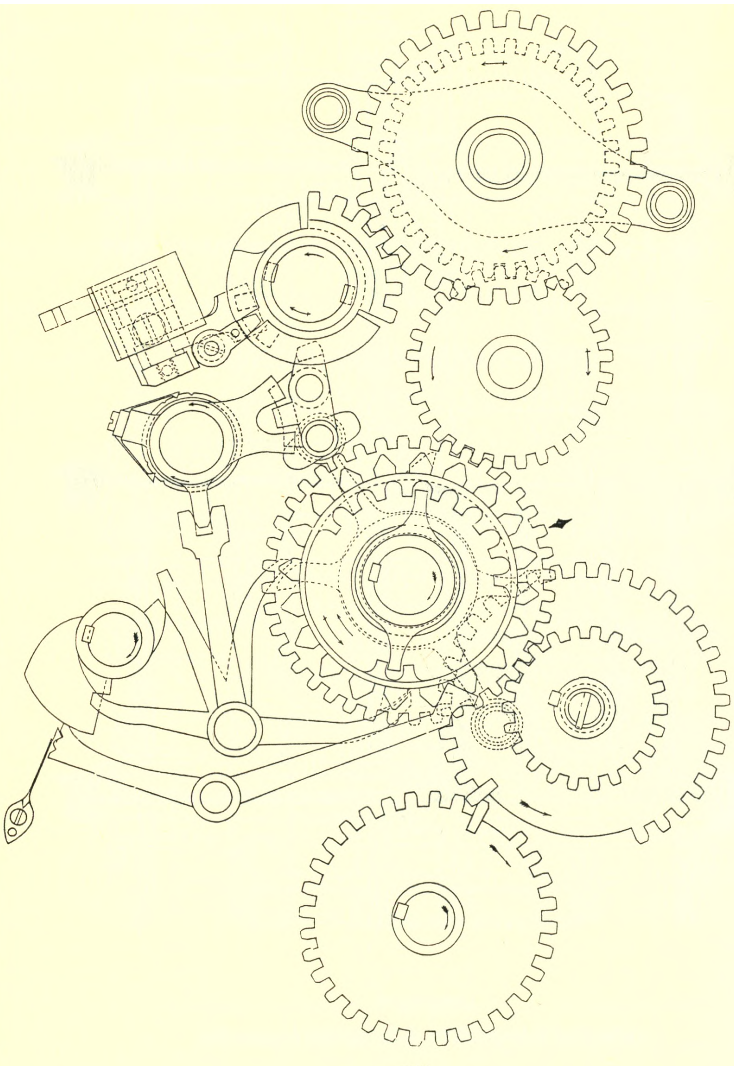

PLATE 11

Plan of mechanism for carrying the tens (by anticipation), connected with long pinions

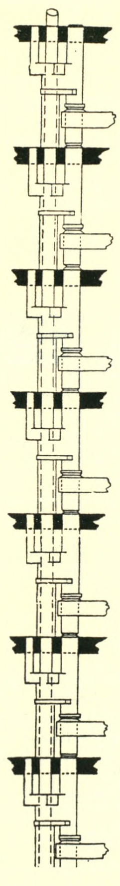

PLATE 12

Section of the chain of wires for anticipating carriage

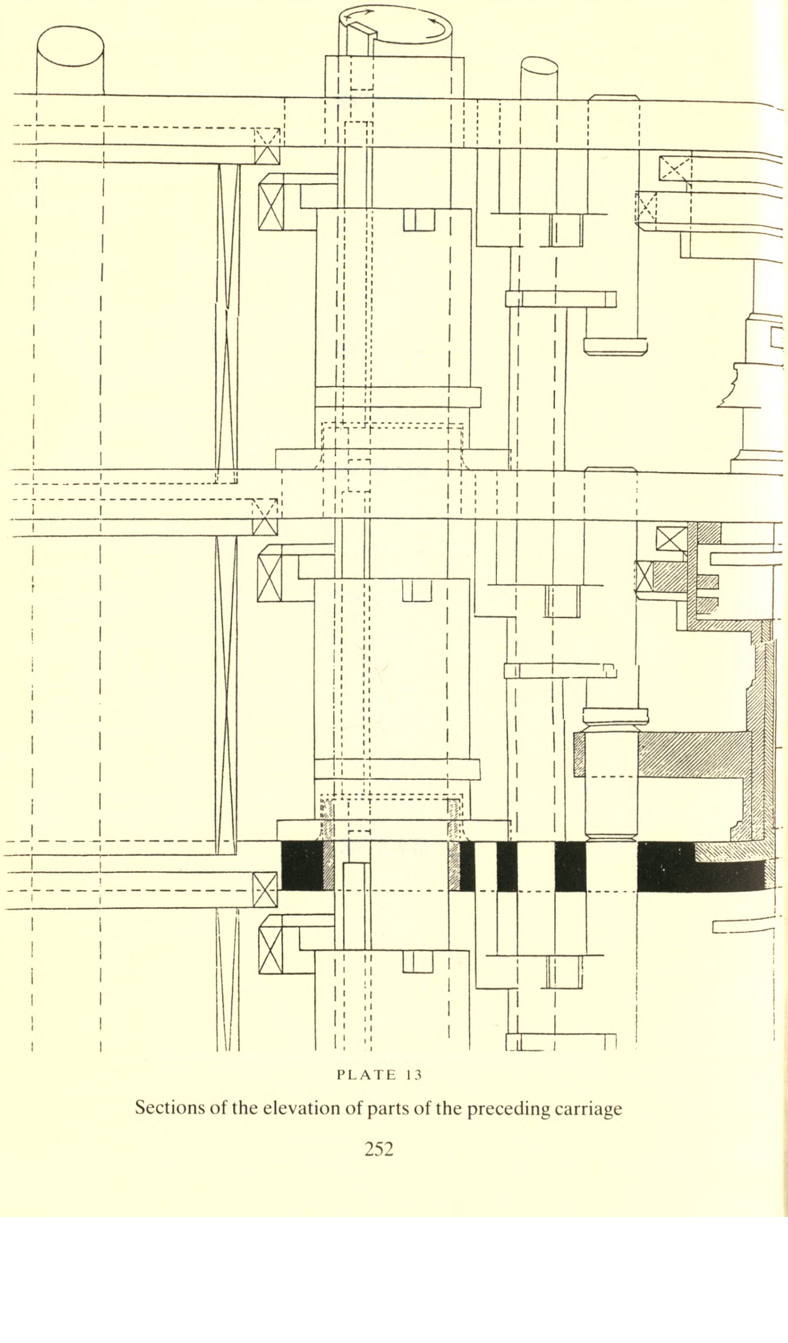

PLATE 13

Sections of the elevation of parts of the preceding carriage

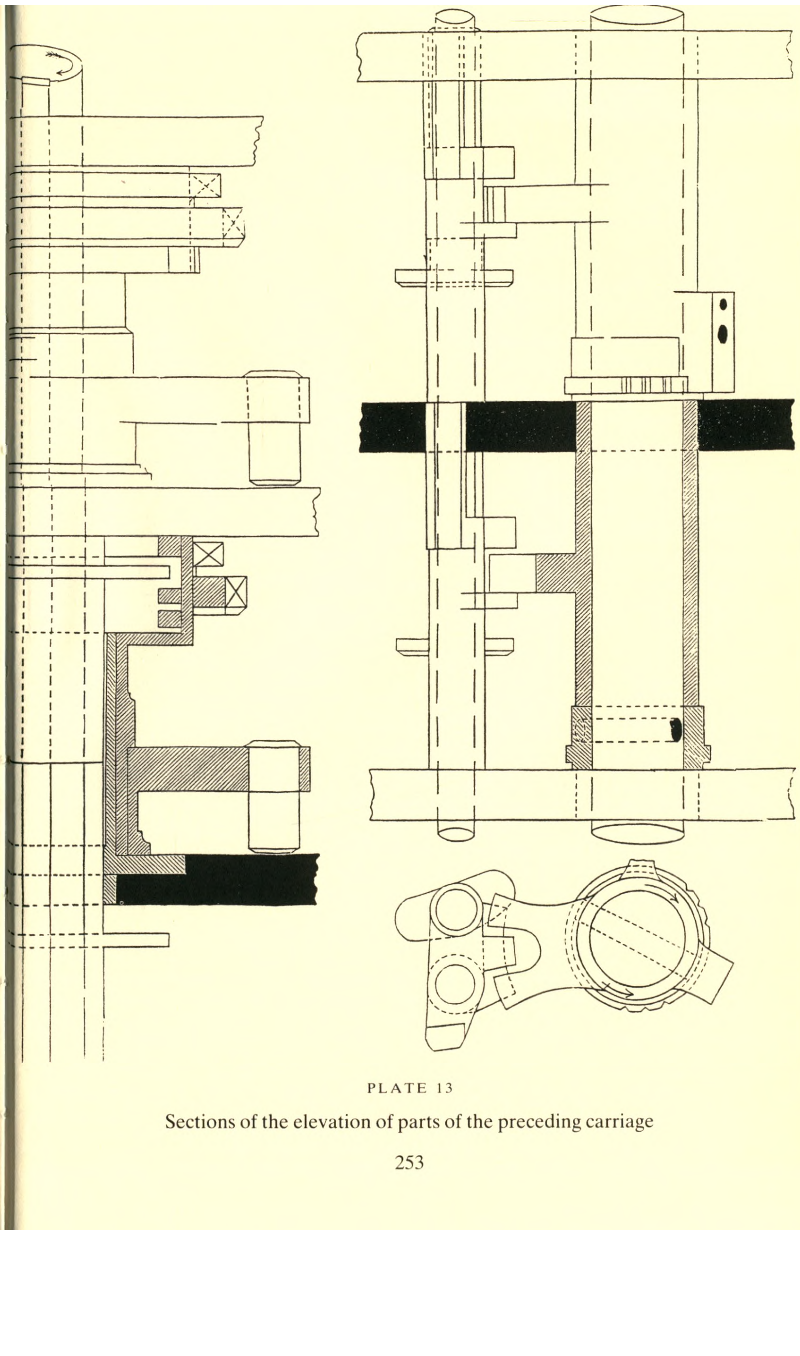

PLATE 13

Sections of the elevation of parts of the preceding carriage