Introduction to Stochastic Processes and Computer Simulation, CSc 85200

Homework Assignment 4

Problem 1 Consider a M/GI/1 queue with Poisson arrivals of rate λ and iid service times $\{S_i\}$ with distribution $\Gamma(3, 0.25)$. The goal is to estimate the stationary average queue length θ. For each simulation model, namely the discrete-event based model and the Petri-net model, do the following. [*Hint:* You may re-use your code from Series 3. If you write your code carefully you should be able to just change a subroutine and use common modules for both models].

(a) Let $\alpha = 0.05$. Use an adaptive algorithm to stop the simulation so that the approximate confidence interval has precision $\epsilon = 0.1$. Assume that you do not know the mean and variance of the service distribution (that is, your program could run by reading consecutive service times from a file with historical data, for example). Explain your choice of the algorithm (independent runs, batch means, discarding of "warm-up" period, etc).

(b) Show that the total number of iterations in your simulation model using the stopping rule that you have defined is a random stopping time with respect to the simulation process.

(c) Perform 20 independent simulation runs to estimate the coverage probability for θ. Discuss your results.

(d) Discuss the results of the two different approaches.

Problem 2 Explain how to apply each of the following methods for variance reduction for the problem above when implementing the Petri net model for simulation:

(a) Antithetic random variables.

(b) Control variable (what variable do you propose to use for the control?)

(c) Perform the simulations with the added methods using the same stopping criterion and record the CPU times for the 20 replications. Compare your results with those of the previous problem.

Problem 3 There are N individuals in a population, some of whom have a certain infection that spreads as follows. Contacts between two members of this population occur in accordance with a Poisson process of rate λ. When a contact occurs, it is equally likely to involve any of the $\binom{N}{2}$ pairs of indivudulas. If a contact involves an infected and a healthy individual, then with probability p the non-infected one becomes infected. Once infected, an individual remains infected throughout. we assume no deaths, and no spontaneous infection. Let $X(t)$ denote the number of infected individuals.

(a) Show that $\{X(t), t \geq 0\}$ is a continuous time Markov Chain (CTMC).

(b) Specify its type (is it a Poisson process?)

(c) Starting with a single infected individual, what is the expected time until all members of the population are infected?

Problem 4 You have decided to do consultation for modeling, simulation and optimization. You offer various research services: modeling, statistical analysis, optimization, software development, etc. There are N such research stages and they always follow a specific order: research of type n is always followed by that of type $n-1$, for $N \geq n > 1$. The time (in hours) required to complete stage n follows a distribution F_n of mean μ_n. Potential clients arrive according to a Poisson process of rate λ, and you take the job only if you are free at the time of arrival of the client. If you are already working on a problem then you do not take new contracts. Each problem starts at stage n with probability p_n and ends at stage 1, following all intermediate stages.

(a) Under what conditions on $\{F_n\}$ is $X(t)$ a CTMC? Give the rates v_i and transition kernel P_{ij}.

(b) Suppose that the distributions $F_n, n = 1, \ldots, N$ are not memoryless. You assume that you will charge c dollars per hour of work. Specify the regeneration points of the process and determine the long term rate of profit.

(c) In order to price your services correctly, let us assume that clients are discouraged if c is too big. Specifically, assume that the probability that clients accept your conditions is given by $P_c = (K-c)/K, 0 \leq c < K$. If $c \geq K$ then $P_c = 0$ (no client will hire you). Use $K = 2, \lambda = 2$, and $M \stackrel{\text{def}}{=} \sum_n p_n (\sum_{i=1}^n \mu_i^{-1} = 1$. Determine the new long term profit rate $R(c)$ and find the optimal value of c.

Problem 5 Variance reduction via conditioning and via IS: to come.