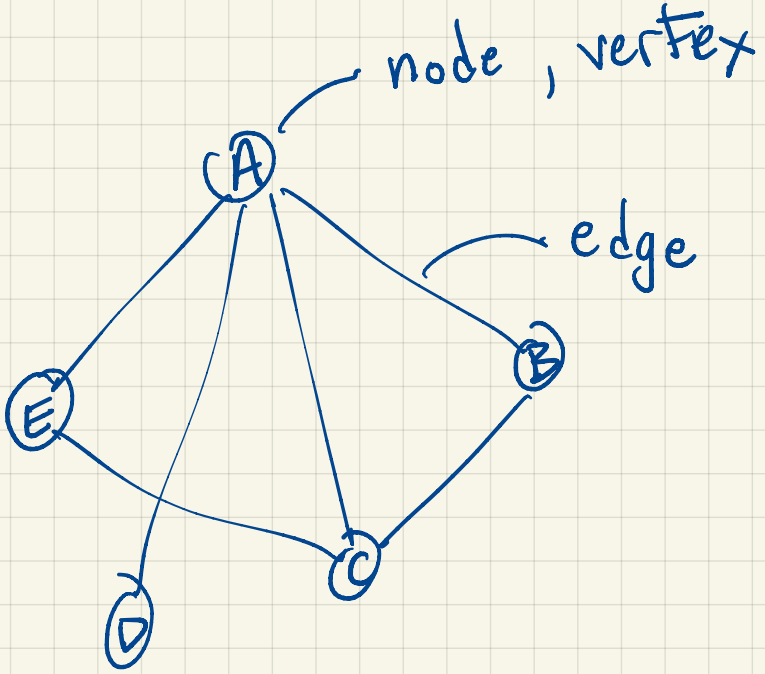
Few things about graphs

Pairwise relation

vertices

edges

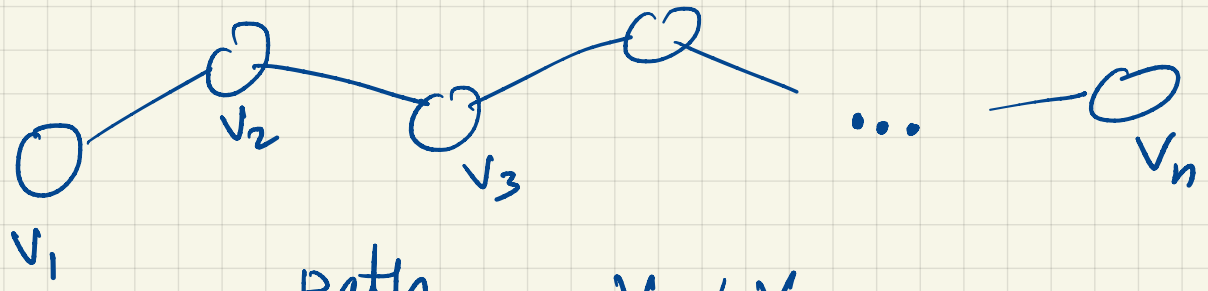


$V = \{A, B, C, D, E\}$

$E \subset V \times V$. For instance $(A, D) \in E$

$(D, A) \in E$

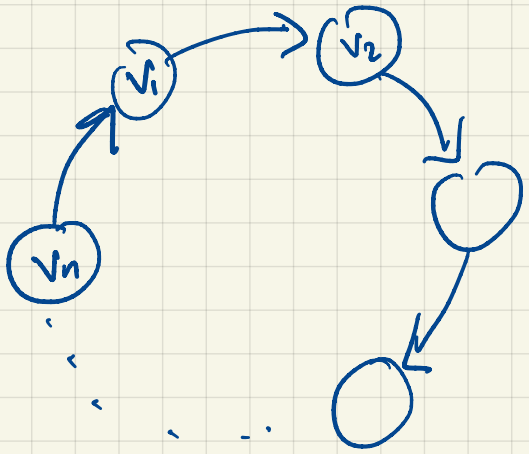
all edges are different



path $\quad v_i \neq v_j$

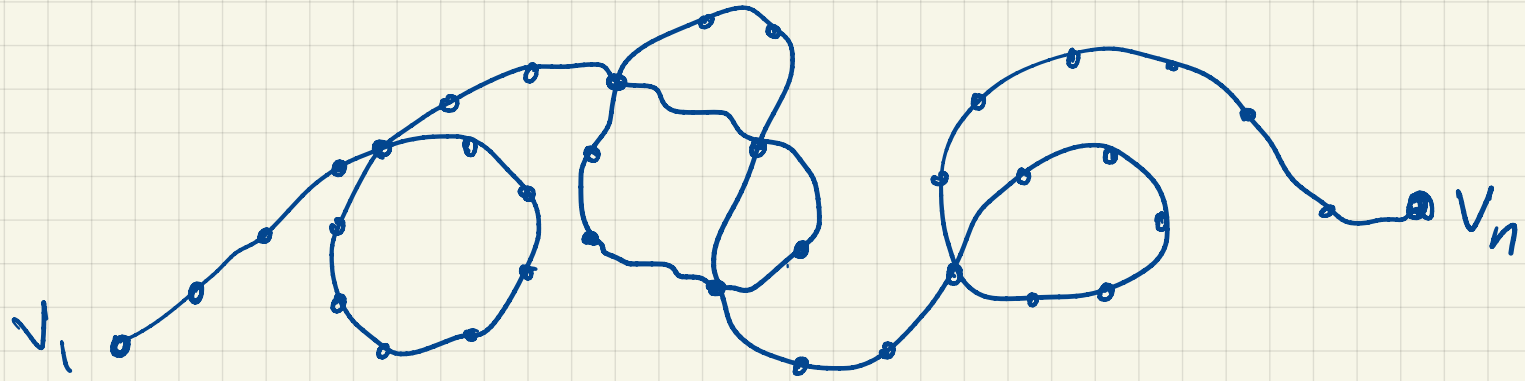
$(v_i, v_{i+1}) \in E$

If $v_1 = v_n \Rightarrow$ path is a cycle



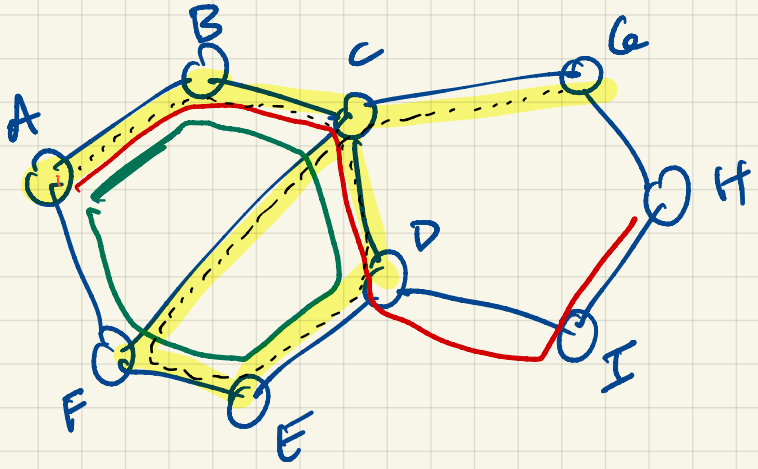
A, B, A not a cycle

(A, B) and (B, A) same edge



"walk"

$V_1 = V_n$ "closed walk"



Path

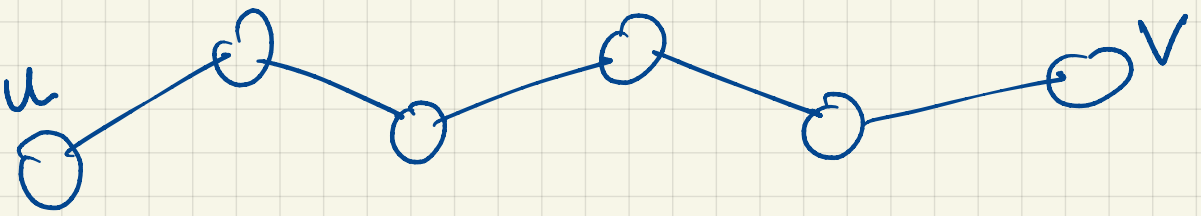
A, B, C, D, E, H

cycle

A, B, C, D, E, F, A

A, B, C, D, E, F, C, G

walk

Path: 

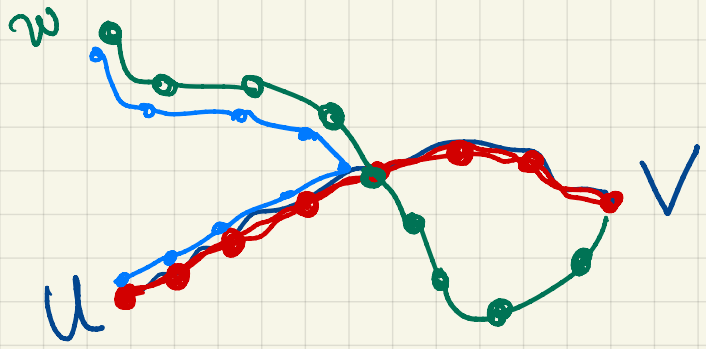
$u \rightsquigarrow v$: there is a path from u to v

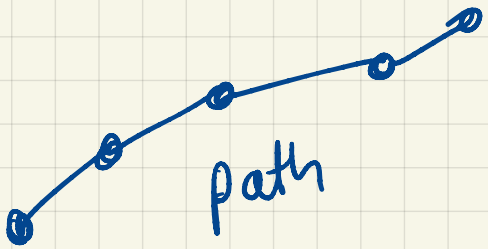
Path is an Equivalence Relation.

Reflexive: $u \rightsquigarrow u$ (empty path)

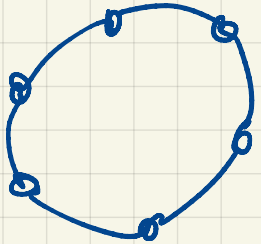
Symmetric: $u \rightsquigarrow v \Rightarrow v \rightsquigarrow u$

Transitive: $u \rightsquigarrow v \wedge v \rightsquigarrow w \Rightarrow u \rightsquigarrow w$

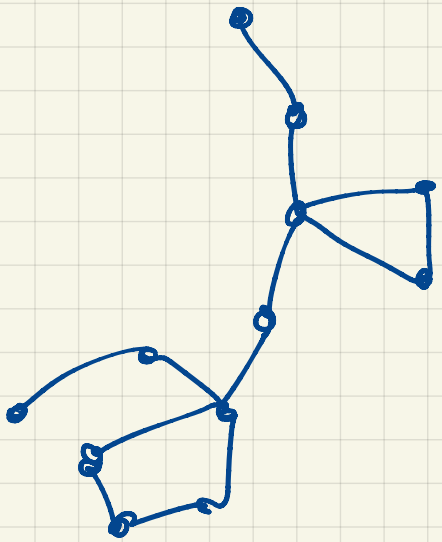




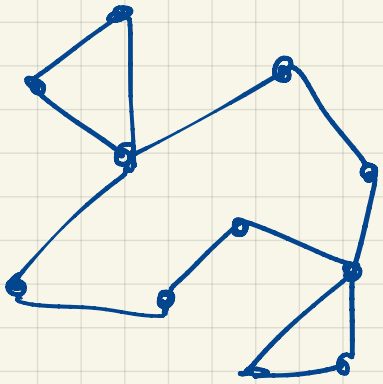
path



cycle

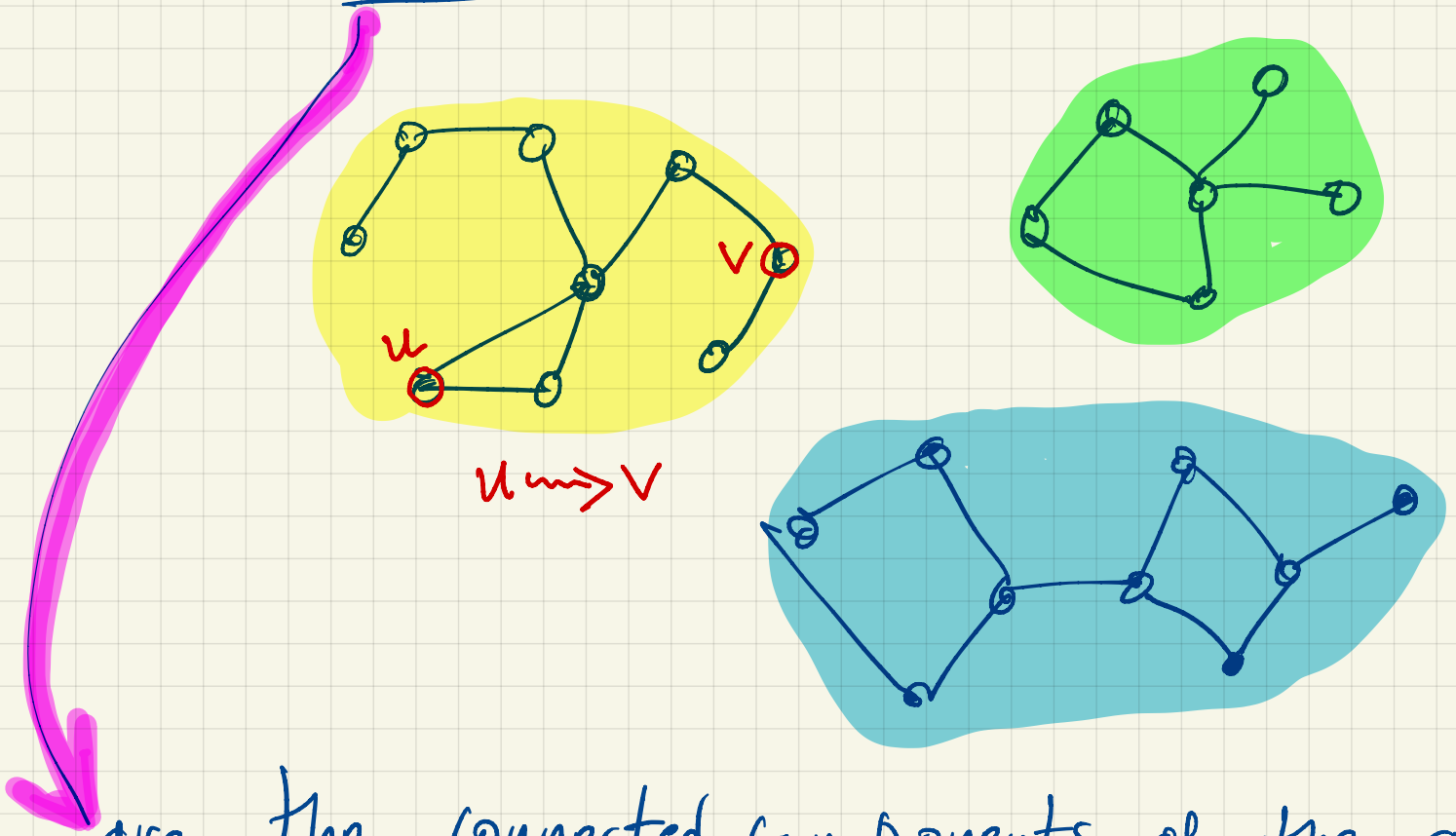


walk



closed walk.

Classes of equivalence under path relation $\rightsquigarrow$.

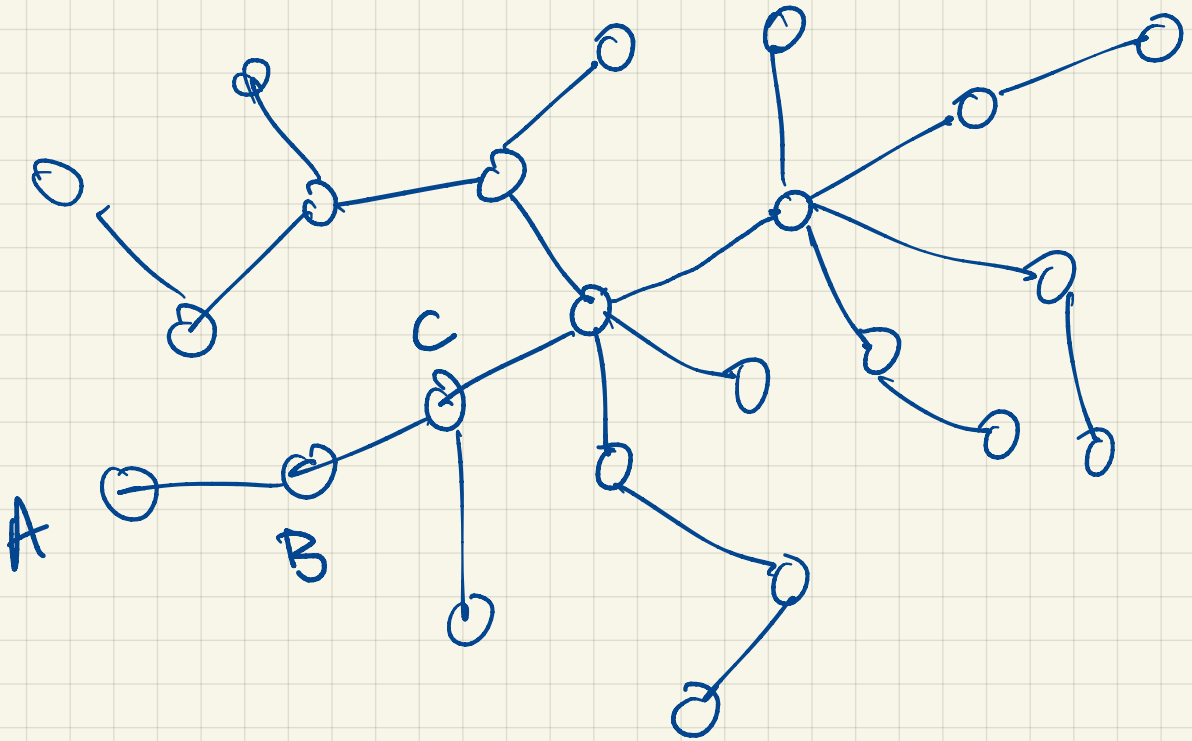


are the connected components of the graph.

A Graph is Connected if it has 1 Connected component.

Trees

Tree is a graph that is Connected and has NO cycles.



A, B, A : Not cycle

A, B, C, B, A: Not cycle

Definition of Tree:

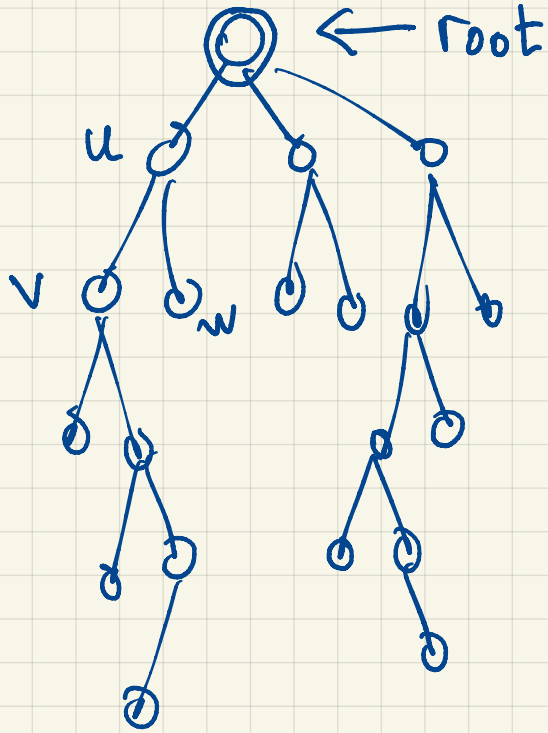
1) Connected & cycle-free

2) Connected but removing any edge disconnects it
   [minimally connected]

3) Cycle-free but adding any edge between 2 existing vertices creates a cycle.
   [maximally cycle-free]

$$\text{Tree} \implies \#\text{edges} = \#\text{vertices} - 1$$

Rooted tree: make 1 vertex a root.

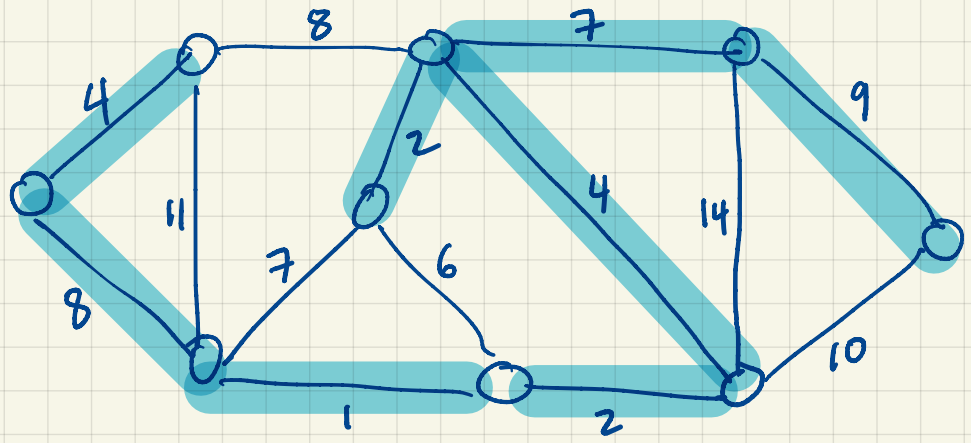


v and w are the "children" of u

u is "parent" of v

root has no parent

Spanning Tree:

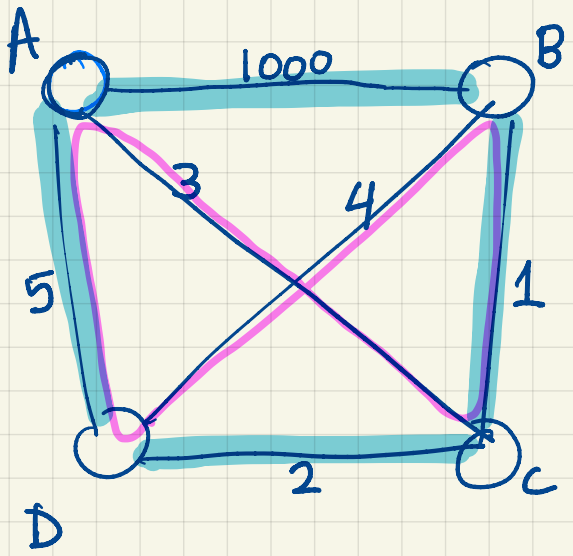


Find a spannig tree with minimum cost.

Cost of tree: sum of costs of its edges.

Be Greedy

Pick the edge with the smallest cost and add it if it does not create a cycle.

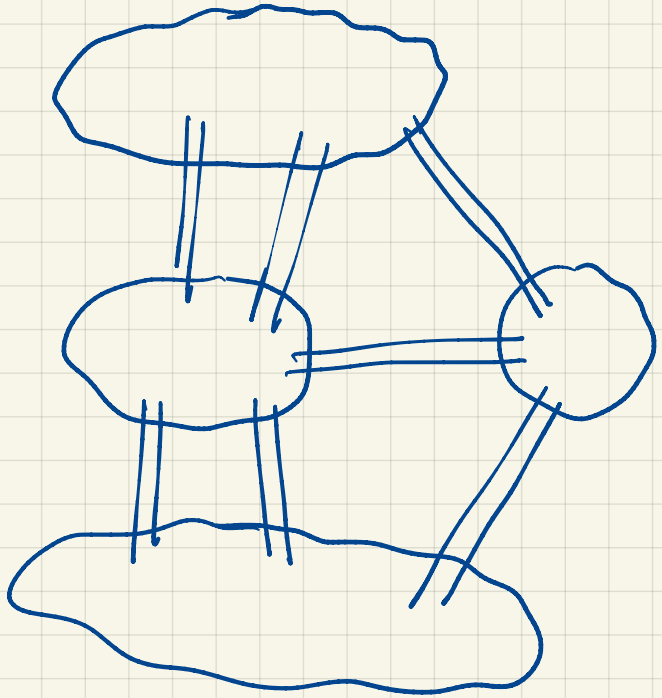


Traveling salesman:
Find cycle that visits every vertex once and has minimum cost.

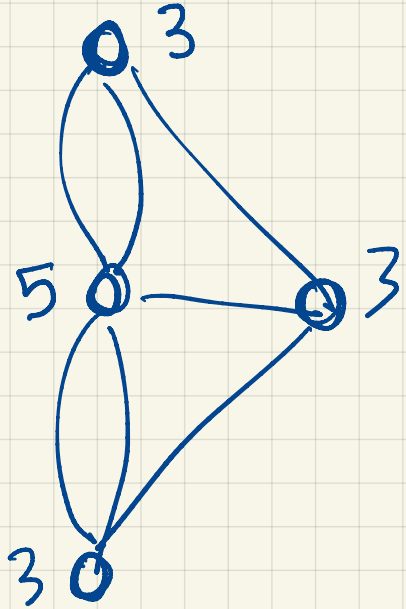
Cost of cycle is sum of costs of its edges.

Greedy does not work!

Euler tour



Bridges of Konigsberg



multi-graph

Degrees must be even