CSCI 150 Discrete Mathematics
Homework 11

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All graphs are assumed to be undirected.

1. Is there a graph that has the sum of degrees of its vertices equal to 13? If Yes, provide one. If No, explain why.
2. Provide two graphs that don't look the same, but have the same set of degrees.
3. In a planar graph, define the degree of a face to be the number of edges that we encounter on a closed walk of its boundary. Let d_f be the degree of face f. What can we say about $\sum_f d_f$? (this is the sum of all face degrees)
4. In a planar graph, s faces have degree 4 and t faces have degree 3. Both s and t are positive integers. We also know that all vertices have degree 3. Using the Handshake Lemma, Euler's formula, and the idea of the previous exercise, show that the graph has exactly 5 faces (solve for s and t).
5. Prove that if a graph with n vertices is disconnected, then $|E| \leq \binom{n-1}{2}$. *Hint*: if the graph is disconnected, then partition the vertices into two sets and argue the maximum number of edges in each set of vertices.
6. A double star tree is a tree with exactly two nodes that have degree > 1. How many double star trees are there if we have n vertices. (we are only concerned with the structure, the vertices are not labeled)

7. A $(k, l) - \textit{dumbell}$ graph is obtained by taking a complete graph on k labeled vertices and a complete graph on l labeled vertices, and connecting them by a single edge. Find the number of spanning trees of this graph.

8. Prove that a graph with n vertices and m edges has at least $n - m$ connected components. Do this by induction on m, so start with $m = 0$ as a base case. For the inductive step, pick a vertex and delete all its edges and reason about the resulting graph. Then add back the deleted edges.