CSCI 150 Discrete Mathematics
Homework 4

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Exercises

1. You have 4 identical balls, and you want to place them in 10 different containers. In how many ways can we do that if we need to satisfy the condition that at least one container must have at least 2 balls? *Hint*: Solve this using these steps:
   - First, count the possibilities that do not satisfy this condition (i.e. every container has at most 1 ball).
   - Then, count the total number of ways of placing 4 identical balls into 10 different containers.
   - Finally, subtract one from the other.
2. You have 10 candies, 2 of which are identical. You want to eat one after breakfast, one after lunch, and one after dinner (childish I know...) Assume that this order of eating the candies is important. In how many ways can you eat 3 candies? *Hint*: Solve this using these steps:
   - First, count the possibilities in which all candies are different (so work with 9 candies).
   - Then, count the possibilities in which 2 of the candies are identical.
   - Finally, add them up.
3. How many integer solutions are there for $x + y + z + w = 15$ if $x \geq 3$, $y > -2$, $z \geq 1$, $w > -3$? (make sure you take care of $\geq$ and $>$).

4. There are 14 men and 9 women. They are to be seated on 23 chairs in a row such that no two women sit next to each other. How many ways are possible? (*Hint*: we did something similar with bits).

5. What is the coefficient of x^3y^2 in $(x + y + 2)^{10}$?

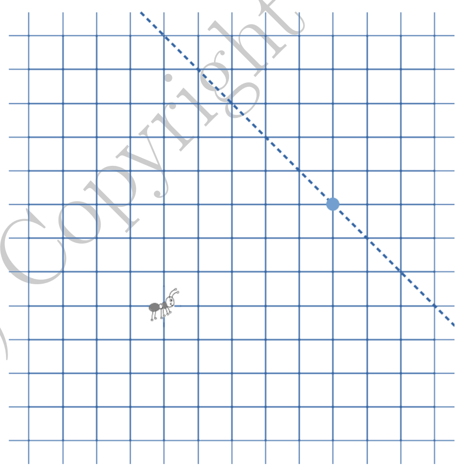
6. Find

$$\binom{100}{0}\left(\frac{1}{3}\right)^{100}6^0 + \binom{100}{1}\left(\frac{1}{3}\right)^{101}6^1 + \ldots + \binom{100}{100}\left(\frac{1}{3}\right)^{200}6^{100}$$

Hint: This form is not the binomial theorem yet!

Problem

An ant sits on an infinite grid at $(0, 0)$ as shown below. There is also food indicated by a dot at $(5, 3)$. The ant is not aware yet of the oblique line (that's Broadway of the ant city), but she will be in part (e).



(a) Assume that the ant senses where the food is and will start moving towards it by only making UP and RIGHT moves. How many possible paths are there that will bring the ant to the food? *Hint*: think of a bijection from the set of paths to some special set of binary words (what is that set?).

(b) In this version of the story, the ant has no idea where the food is. So she started to move from $(0, 0)$ by making a sequence of random UP, DOWN, LEFT, and RIGHT moves, only to find herself back to her original position after 8 moves. Although she was disappointed, she thought: "How many possible paths of length 8 will bring me back to my original position?" Well, she did not really think that, but I did. Please count those using the following strategy:

- First, divide the paths into disjoint categories, depending on the number of UP moves in the path.
- Then observe that the number of UP moves completely determines the number of all other types of moves!
- Then think of the path as an anagram of length 8 made of U, D, L, and R.
- Finally, use the addition rule.

(c) The ant gained some magical powers (and let's not get into how this happened)! Now she can move according to the following function:

$$f : \mathbb{Z}^2 \to \mathbb{Z}^2$$
$$f(i, j) = (k, l) = (i + j, i - j)$$

So if the ant is at position (i, j), she will next be at position $(k, l) = f(i, j)$, and so on.

- Give convincing arguments that f is one-to-one but **not** onto. To show one-to-one, use our standard strategy by starting with $f(i, j) = f(i', j')$ and concluding that $(i, j) = (i', j')$.
- Show that despite her magical powers, there is only one starting position, except position $(5, 3)$ itself, that will eventually bring the ant to the food. *Hint*: think backwards.

(d) Aimless, the ant decided that she will make a total of 8 moves with at least one UP move and at least one RIGHT move. She also decided that she will first make all the UP moves, then all the LEFT moves, then all the DOWN moves, then all the RIGHT moves. How many paths are possible?

(e) If the ant will only make UP and RIGHT moves, but will also decide whether to deposit pheromones every time she makes a RIGHT move, in how many ways (including pheromone pattern) can she reach Broadway (the oblique line)? *Hint 1*: Think about the number of ways to reach each of the grid points on the oblique line, then use the addition rule. *Hint 2*: The Binomial theorem will be helpful here.