Spectrum map aided multi-channel multi-hop routing in distributed cognitive radio networks

Saptarshi Debroy and Mainak Chatterjee
Department of Electrical Engineering & Computer Science
University of Central Florida, Orlando, FL 32816
Email: {saptarsh, mainak}@eecs.ucf.edu.

***Abstract*—In this paper, we propose a spectrum map aided routing protocol for a distributed cognitive radio network. We assume the presence of dedicated sensors that capture the spatio-temporal spectrum usage statistics to create the radio environment map. We exploit the map to find not only the best hops along a route but also the best available channel in terms of the expected performance. Through the use of *edge* nodes, which lie in the intersection of more than one sensors' domain, inter-domain routing is facilitated. The selection of each hop, the channel to be used per hop, and the transmitting power to be used considers i) protection of primary receivers, and ii) maximization of desired performance metric. In this context, we propose a novel power control mechanism that computes just enough power to maintain the desired signal-to-noise ratio for secondary communication but at the same time protects the primary receivers in the vicinity. We analyze and compute the probability of network connectivity by finding the minimum spanning tree of the graph formed by the over-lapping domains. Through simulations, we show how the proposed routing scheme works in terms of route capacity, connectivity of the network, reachability among the nodes, and number of primary receivers protected.**

I. Introduction

Cognitive radio networks (CRN) are envisioned to address the spectrum scarcity problem by allowing secondary nodes to exploit the spectrum availability in an opportunistic manner. Since spectrum availability is space and time variant, selection of the best channel between a given pair of transmitter and receiver is very crucial. When some information has to be routed over multiple hops, sophisticated routing algorithms are needed that can not only find the best route but also find the best channel to use at every hop.

The main challenges for routing in distributed CRNs over conventional ad-hoc networks are: spectrum-awareness, maintenance of routes from sudden appearance of primaries, and protecting primary receiver from harmful interference of secondary communication. Designing efficient routing solutions for multi-hop CRNs requires a tight coupling between the routing module and the component responsible for managing spectrum availability such that the routing module can be continuously aware of the surrounding physical environment. There are two different ways to achieve this: a database or radio environment map, e.g., TV whitespace database, or through distributed sensing mechanisms at secondary locations. The recent trend is more towards 'query and use' which is facilitated through radio environment maps [1], [2], [3] and spectrum databases [4]. Therefore, it is essential that cognitive routing protocols leverage such up-to-date spectrum information in building efficient and robust protocols. Moreover, an important regulatory constraint is that the primary receivers must be protected from any kind of harmful interference caused by secondary communications. Such primary protection can be made more effective by the use of spectrum maps since the spectrum usage scenario of a region can be made known to the routing entities. Moreover, reference from a spectrum map can help in building a routing scheme which can couple the legacy ways of assessing the route quality with novel measures on path stability, spectrum availability.

Cognitive routing protocols are broadly categorized into two main classes: full spectrum knowledge [5], [6] and local spectrum knowledge [7], [8]. Although these works make valid contributions on routing in secondary networks, they fail to guarantee primary receiver protection and maximize desired routing performance metric at the same time. Most of the previous work either ignore the hidden and exposed terminal characteristics of primary receivers or assume a over-conservative primary contour protection scheme which considerably decreases the achievable secondary throughput.

In this paper, we propose a novel power-controlled multi-channel, multi-hop secondary routing scheme which takes advantage of a spectrum map. The routing protocol is targeted for routing packets among low-cost secondary nodes with no sensing capability and no coordination among themselves. For such nodes, sensing capability is de-coupled from the device. Our scheme employs some sensors to sense and build a spectrum map and use the spectrum availability information to identify best possible route, i.e., intermediate hops, and the best channel to use. With the help of the map, the sensors compute the optimal power for each node on each channel which protects any primary receivers in the vicinity of the node, and maximizes the link capacity at the sam time. The routing scheme is flexible enough to take different measures according to the distance between the source and the destination and uses a novel *selective flooding* technique to limit the overhead of route request forwarding. We analyze the system under different scenarios and define conditions for connectivity. Through a rigorous simulation model, we show the correctness of the analysis, and investigate the nature of connectivity of the network. The results show that when power

978-1-4799-4912-0/14/$31.00

control is used to protect the primary receivers, the reachability among nodes decreases but not to the extent that there are no routes between the source and the destination. The results also show how number of protected primary receivers changes with the density of primary receivers in the network and the distance between the source and the destination nodes.

II. System Model

We consider a geographic region consisting of primary networks, secondary networks, and a collection of sensors that periodically sense primary activity and create a spectrum map.

A. Spectrum Map

A spectrum map is a 3-dimensional representation of spectrum utilization in a geographical region either using both theoretical models or real-world data logs [1], [2], [3]. These models/techniques allow secondary networks to compute or predict the spectrum usage at arbitrary locations. Most of these spectrum map construction techniques are flexible enough to be used for varied cross-layer secondary network services ranging from resource allocation, MAC design to routing schemes. Although our proposed routing scheme can utilize any of such map construction techniques, their design and implementation specifics are beyond the scope of this paper.

B. Primary network

We consider a primary network consisting of a collection of licensed transmitters and receivers which operate independently of secondary nodes. We assume that the primary transmitters are Poisson distributed. These primary transmitters operate on a pre-defined spectrum band and follow a well-known ON-OFF model for transmission pattern. We assume that the signal strength diffuses isotropically in the environment and is received at any location with transmission power P multiplied by a loss factor due to isotropic dispersion and absorption in the environment.

C. Secondary network

Secondary nodes seek to access the the channels not being used by the primaries. The secondary network has two components: intelligent sensors in the control plane and unintelligent secondary nodes in the data plane.

Sensors: The sensors are deployed in the area of interest either at strategic locations or randomly depending on the technique used for the construction of the spectrum map. The sensors' responsibilities are broadly two-fold: spectrum map creation and route discovery. The sensors periodically sense the primary activities, share their information among themselves and build the spectrum map. Route discovery includes receiving route requests (RREQ) from secondary nodes, finding routes to the destination or the edge nodes, and caching potential routes. Sensors communicate with each other using a control channel. The same control channel is used to communicate with the secondary nodes as well. Each sensor has a transmission range of r_s, the area under which is called a *domain* and secondary nodes within the domain are under the purview of that sensor.

Secondary Nodes: Secondary nodes are deployed irrespective of primary and sensor locations as a two dimensional Poisson point process. These secondary nodes have no sensing capability and are instructed by the sensors to use a particular channel intended for a particular destination. Like a primary transmitter, a secondary node is a transceiver with no fixed transmission range. The connectivity among secondary nodes is a function of the availability of free channels, secondary transmission power, path loss and other propagation characteristics like shadowing and fading. Secondary nodes under the purview of a single sensor are called *non-edge* nodes while nodes lying in the overlapping regions are called *edge* nodes. Although the edge nodes reside in the overlapping region, they are associated to only one sensor at a time. However, they can cache other sensors' IDs for use in route discovery. Secondary nodes outside the disc of any sensor is an uncovered node.

D. Design Optimizations

The routing protocol is designed to optimize two aspects: guaranteeing protection of all the primary receivers along a secondary route and maximizing desired route performance metric. For this work, we chose achievable route capacity as the desired metric. Although, the maximum achievable capacity along a route is the minimum of all the link capacities called *bottleneck capacity*, we argue that maximizing the overall route capacity automatically maximizes the bottleneck capacity of the route. It is to be noted that the proposed routing scheme is capable to maximize any route quality metric, e.g., throughput, route stability, spectrum availability etc. or a combination of these metrics.

III. Proposed Routing Scheme

Route discovery is initiated when a secondary node sends a route request (RREQ) to its associated sensor on the control channel. The proposed routing scheme employed by the sensor determines three aspects of routing: next hop selection, channel selection, and secondary power control for primary receiver protection.

A. Route discovery

A route from source to destination can be of two kinds depending on their relative locations: intra-domain and inter-domain. We will first discuss intra-domain routing and then explain how inter-domain routing is treated as a collection of intra-domain routing.

1) Intra-domain routing: A sensor upon receiving the RREQ checks whether the destination is within its domain. If so, for each source node i, the sensor consults the most recent spectrum map and eliminates all the channels which are occupied. For all the available channels in the spectrum, the sensor calculates $\overline{P}_i^n$ which is the upper bound on secondary transmission power while using channel n so that no primary receivers are interfered on that channel.

Graph creation: We define $\mathcal{P}_i^n = min\{P_{hw}, \overline{P}_i^n\}$ to be the optimum power used on channel n which will maximize the channel performance while protecting the primary receivers on that channel. P_{hw} is the maximum secondary transmission

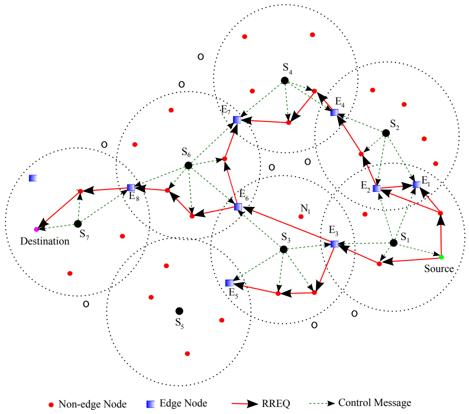


Fig. 1. Inter-domain routing: RREQ forwarding through selective flooding.

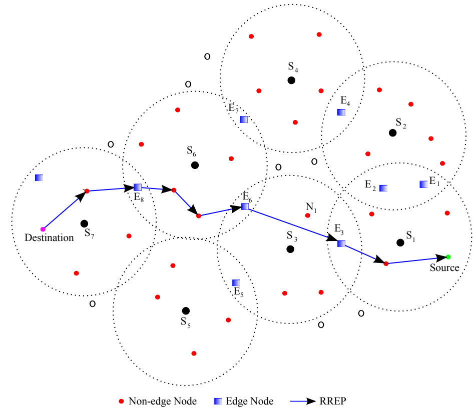


Fig. 2. Inter-domain routing: RREP without sensor involvement.

power due to hardware constraints and is assumed to be the same for all secondary nodes. For every node $j (j \neq i)$ within the domain, if $RSS^n_{ij}/\eta^n_j \geq \delta$, then there exists an edge between nodes i and j for channel n. Here, RSS^n_{ij} is the received signal strength at node j on channel n estimated by the sensor when node i transmits with power $\overline{P}^n_i\}$, η^n_j is the noise on channel n at j from the spectrum map, and δ is the signal to noise threshold for successful secondary communication. The sensor can easily calculate RSS^n_{ij} using any sophisticated pass-loss model; the more sophisticated the model, better is the estimation. Therefore, for all such n channels between i and j, there exists an edge e^n_{ij} from i to j which is associated with a cost ζ^n_{ij}. Although the cost function can be designed as a complicated combination of classical and novel route quality metrics, we design the cost as a reciprocal of the achievable capacity of channel n in order to satisfy the design objectives. Therefore,

$$\zeta^n_{ij} = Blog_2(1 + \frac{RSS^n_{ij}}{\eta^n_j}) \tag{1}$$

By employing any well known shortest path algorithm (Dijkstra's, Ford Fulkerson's etc.), the sensor determines the shortest path between the source and the destination within its domain. The shortest path thus contains the next hop IDs, channel to be chosen for each hop, and secondary transmission power for each such channel. Once the path is determined, the sensor sends the routing instructions to the corresponding nodes along the path on the control channel.

$\overline{P}^n_i$ **calculation:** Evaluating $\overline{P}^n_i$ is an intuitive reverse calculation to protect primary receivers. Let $\overline{d}^n_i$ is the shortest distance from node i to the location where channel n is no longer vacant, called the *safe zone* distance. This distance can easily be measured by the sensor from the spectrum map. Therefore, the disc with radius $\overline{d}^n_i$ with node i at the center has the smallest area where the primary receivers are interference-free on channel n. It is to be noted that this so-called *safe zone* for the primary receivers is independent of the primary receiver distance from the secondary node i. Now if $\overline{RSS}^n_i = f(\overline{d}^n_i, \overline{P}^n_i)$ is the estimated received signal strength at the perimeter of that disc, and κ is the secondary to primary interference tolerance threshold, then to guarantee primary receiver protection,

$$\frac{\overline{RSS}^n_i}{\eta^n_{sz}} \leq \kappa \tag{2}$$

where η^n_{sz} is the noise on channel n at the perimeter of the safe zone. We use a highly sophisticated path-loss model proposed in [9] for the sensors to estimate $\bar{RSS}^n_i$. Using that model, $\overline{P}^n_i$ is given as,

$$\overline{P}^n_i = \frac{\eta^n_{sz} \times \kappa \times 16\pi^2(\overline{d}^n_i)^\gamma}{\lambda^2 d_0^{\gamma-2}} \tag{3}$$

where γ is the average path-loss factor, d_0 is the antenna far field, and λ is the wavelength of light.

2) Inter-domain routing though selective flooding: When the source and destination nodes are not under the same sensor, the idea is to flood the route request in the neighboring domains. Therefore once the sensor determines the need of inter-domain routing, it finds the shortest route from the source to each of the edge nodes currently within its domain. The edge nodes, upon the reception of a RREQ where the edge node itself is not the final destination, try to connect to the other sensor/s and initiate a RREQ. For example for an edge node k having sensor priority $\{S_2, S_5, S_1\}$ means that k is currently covered by S_2 and it also lies within the disc of S_5, and S_1. Such priorities can be based on the sequence of beacons received from the sensors while the edge node is first powered up. Once the neighboring sensor receives the RREQ, it follows the same recursive process of finding a route to the destination or to the edge nodes until the final destination is found. The route discovery scheme employs selective flooding where a sensor does not cater to the same RREQ request through its domain. In Fig. 1, we show how selective flooding works and explain duplicate RREQ scenarios. Such selective flooding considerably decreases the route discovery overhead without compromising the discovery of multiple routes to the destination. We also show the control messages from sensors directing relay nodes along the path. In Fig. 2, we show the RREP packet flow from destination to the source without the involvement of the sensors.

B. Route maintenance

The route maintenance is carried out only by the sensors through route caching as they are aware of the current spectrum usage scenario. Sensors cache only those routes which connect each edge nodes in their domain to all other edge nodes in their domain. This is because those routes connecting the edge nodes are the most popular routes for inter-domain routing and in most cases include subsets of intra-domain routes as well. Sensors use the cached route only when there is negligible change in the spectrum maps from when the route was cached. Secondary nodes lying within such cached routes automatically benefit from such caching.

IV. Mathematical Modeling

We model the routing scheme for a deployment of sensors, each with domain radius r_s in a deterministic grid pattern, equidistant from horizontal and vertical neighbors. This particular orientation is chosen for relative simplicity of mathematical analysis. However, the principles of our mathematical deduction hold true for any deployment of sensors and secondary nodes. We consider a grid of $l \times l$ dimension as our area of interest. The distance d_{ij} between sensors i and j are kept in such manner that every sensor domain overlaps with the four neighboring domains but the overlapping regions of the domains do not overlap with each other. An example of such a deployment is shown in Fig. 3. Note, that for the following deployment $\sqrt{2}r_s \leq d_{ij} \leq 2r_s$.

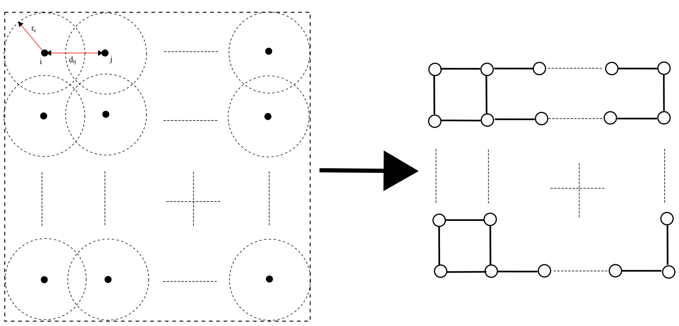


Fig. 3. Deterministic grid deployment of sensors with the overlapping regions and the corresponding mapped graph.

A. Edge node probability

Definition 1: Edge node probability is defined as the probability of any secondary node to be an edge node, i.e., residing in an overlapping region.

For the above mentioned deployment, the total number of overlapping regions, N_{overlap}, is $2\sqrt{\mathcal{N}_{sen}}(\sqrt{\mathcal{N}_{sen}}-1)$, where $\mathcal{N}_{sen}$ is the number of deployed sensors in the grid. The area under each overlapping region is:

$$A_{\text{overlap}} = r_s^2(\theta - sin\theta)$$

where $\theta = 2\tan^{-1}\left(\frac{\sqrt{4r_s^2 - d_{ij}^2}}{d_{ij}}\right)$

Therefore edge node probability can be expressed as:

$$p_{edge} = \frac{N_{\text{overlap}} \times A_{\text{overlap}}}{l \times l} \quad (4)$$

and the expected number of edge nodes is:

$$\text{E[Number of edge nodes]} = \mathcal{N}_{SU} \times p_{edge} \quad (5)$$

where $\mathcal{N}_{SU}$ is the number of secondary nodes in the grid.

B. Connectivity condition

Multi-hop inter-domain routing from any source to any destination needs to satisfy two conditions: i) both source and destination need to be covered, i.e., under some domains, and ii) those domains need to be connected with each other directly or indirectly though other domains. The first condition is fulfilled when the source and destination are any edge or non-edge nodes under the purview of a sensor. A combination of higher $\mathcal{N}_{sen}$ and r_s ensures minimum number of uncovered nodes. However, the second condition is dependent on the overlapping regions of the domains and presence of edge nodes in those overlapping regions. This is because, the edge nodes are essential for inter-domain RREQ spreading. The number and locations of such overlapping regions in turn depend on the deployment of the sensors and their relative orientation. We further investigate the conditions that dictate the connectivity of sensor domains.

Definition 2 (Connectivity Condition): The connectivity condition of any secondary network is defined as the sufficient condition for the existence of at least one path from any domain to all other domains in the network.

We formulated the *Connectivity Condition* by mapping the secondary network into a connected undirected graph with domains as vertices and overlapping regions as the edge between the vertices as shown in Fig. 3.

Definition 3 (Mapped Graph): The graph representation of a secondary network with domains as vertices and overlapping regions as edges is called a mapped graph.

Lemma 1: The connectivity condition for a secondary network is that a secondary network can be called connected if there exists at least one edge node at each of the edges of any one of the minimum spanning trees (MST) of the mapped graph.

Proof: Let $G_{n\times n}$ be a mapped graph of any above mentioned sensor deployment with n^2 nodes. Let us assume that it has $\tau(G_{n\times n})$ MSTs. Then each such MST has (n^2-1) edges that connect all the nodes. If we remap the MST into a sensor deployment then it represents a network of minimum number of overlapping regions connecting all domains. Presence of any edge node in each of such overlapping regions will guarantee at least one path from all covered nodes to all other covered nodes in the secondary network. Thus the total number of overlapping regions is a measure of minimum number of edge nodes required for a network to be connected. Hence proved.

For a secondary network deployment shown in Fig. 3, there are $\mathcal{N}_{sen}$ sensors; hence $\mathcal{N}_{sen}$ domains. Therefore the mapped graph of the network will look like a $\sqrt{\mathcal{N}_{sen}} \times \sqrt{\mathcal{N}_{sen}}$ grid. The number of edges in any of the MSTs of such a mapped graph is the count of minimum number of edge nodes required for the corresponding secondary network to be connected. If τ is the total number of possible minimum spanning trees in such a grid, then each MST contains $\big(\mathcal{N}_{sen}-1\big)$ edges.

Therefore, the probability of connectivity condition is given as:

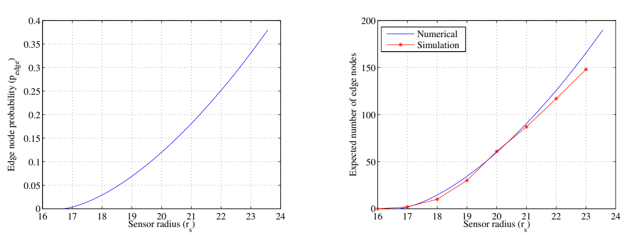
$$p_{conn} = \tau \times \text{Prob}\{Z_1 \geq 1, Z_2 \geq 1, \cdots Z_{\mathcal{N}_{sen}-1} \geq 1 \mid Z_1 + Z_2 + \cdots + Z_{\mathcal{N}_{sen}-1} \leq \mathcal{N}_{SU}\} \quad (6)$$

where Z_i is the random variable denoting the number of edge nodes in the ith edge of the mapped graph. For a $\sqrt{\mathcal{N}_{sen}} \times \sqrt{\mathcal{N}_{sen}}$ square grid, $\tau \approx 3.209^{\mathcal{N}_{sen}}$ for $\mathcal{N}_{sen} \to \infty$ [10], [11]. In Section V, we will evaluate p_{conn} for a given secondary deployment.

V. Simulation Results

We conduct extensive simulation experiments in C and MATLAB to gauge the performance of the proposed multi-channel, multi-hop routing protocol. The primary transmitters are deployed in a 100×100 area using the deployment model used in [12]. We consider varying number of channels from 5-30 with 1 MHz bandwidth each. The primary channel selection at any point is random. Each primary transmitter has a transmission power of 50 Watts and the primary detection threshold is kept at -116 dBm to conform with TVWS standard [13]. Nine sensors are deterministically deployed in a grid patter as discussed in Section IV. The secondary nodes are deployed following a Poisson Point Process to ensure that their locations are not inter-dependent. The secondary transmission power is kept at 100 mW. We used the highly sophisticated path-loss model proposed in [9] which mimics the real life propagation characteristics in an urban macro-cell. The co-channel interference threshold for primary receivers caused due to secondary communication is kept at -80dBm which is the interference threshold for analog TV to digital TV.

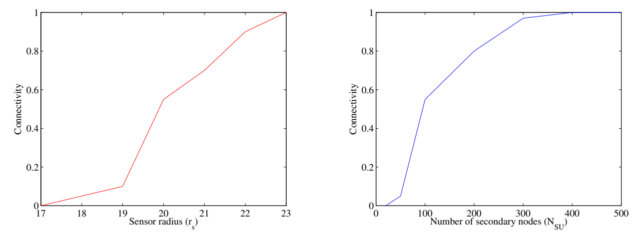
Edge Node Probability: In Fig. 4(a), we show the nature of probability of edge nodes (p_{edge}) with varying radius, r_s. The value of r_s is varied from $\sqrt{2}r_s \leq d_{ij} \leq 2r_s$ (16 to 24) as this is the range where the domains start overlapping but the overlapping regions do not overlap with each other as discussed in Section IV. We see that within this range of r_s, p_{edge} increases rapidly. In Fig. 4(b), we plot Eqn. 5 against the same range of r_s and compare the numerical and simulation values. The simulation results closely match with the numerical trend from Eqn. 5 which in turn validates the mathematical analysis.



(a) (b)

Fig. 4. (a) Edge node probability with varying sensor radius, (b) Expected number of edge nodes for simulation and numerical models.

Connectivity: In Fig. 5(a), we show how the probability of connectivity p_{conn} varies with r_s. Here N_{SU} is kept constant at 100. We see as p_{edge} increases with r_s, so does p_{conn}. The secondary network reaches complete connectivity at r_s = 23, i.e., at this point at least one edge node is present on each edge of at least one of the MSTs of the mapped graph. The nature of p_{conn} with varying N_{SU} is shown in Fig. 5(b) with r_s = 20. We see that with a denser network of secondary node the connectivity increases. With 300 secondary nodes, the network is fully connected, i.e., there is at least one route from each domain to each other.



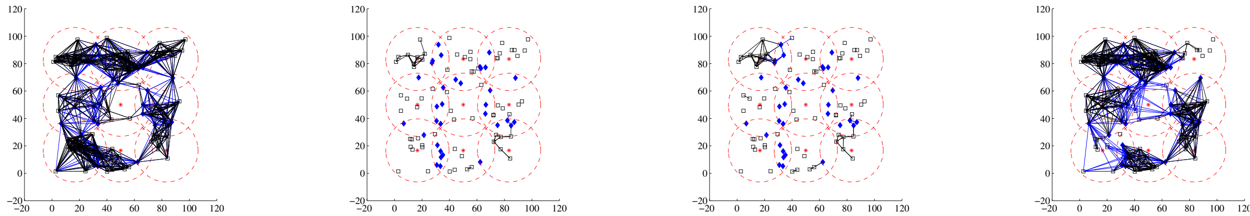
(a) (b)

Fig. 5. Probability of connectivity condition with (a) with varying sensor radius, (b) with varying number of secondary nodes.

Reachability: In figures 6(a)-6(d), we show the reachability of secondary nodes with and without power control. Edge and non-edge connections are shown in different colors. In figures 6(a), we show the reachability with no power control, i.e., the scenario does not take into account the primary hidden terminal problem and thus does not protect any primary receivers that may be present in the communication range. It is to be noted that in this scenario each link is bidirectional, i.e., a link represents both nodes are reachable from each other. In figures 6(b) and 6(c), we observe much less reachability when power control is applied on the secondary nodes. We see that in order to protect possible secondary receivers, the secondaries had to use much less power thus the reachability decreases considerably. With more channels in Fig. 6(c), the reachability increases marginally. However in Fig. 6(d), when we change the minimum SINR requirement to from 50dB to 15dB, which is more comparable with commercial wireless standards, we see reachability increasing even with power control. However, in this case, unlike no power control, each link may or may not be bidirectional. This is due to the fact that it is not always true that both nodes connecting the links will not cause interference to primary receivers.

Routing: We show the nature of bottleneck capacity to average route capacity in Fig. 7. With higher r_s, network connectivity increases, thus the proposed routing scheme identifies better routes with higher bottleneck capacity. With inter-domain routing, the probability of a finding a lower capacity link increases, thus we observe reduction in bottleneck capacity.

In figures 8(a) and 8(b), we show how the number of protected primary receivers for power control changes with the distance between the source and destination node and primary receiver density. In Fig. 8(a), we show that the number of primary receivers protected increases as the distance between the source and destination increases. This is due to the fact that with no power control, more primary receivers are interfered



(a) (b) (c) (d)

Fig. 6. Reachability (a) without power control for 5 channels (b) with power control control for 5 channels (c) with power control control for 20 channels (d) with power control control for 5 channels and SINR threshold 15 dB.

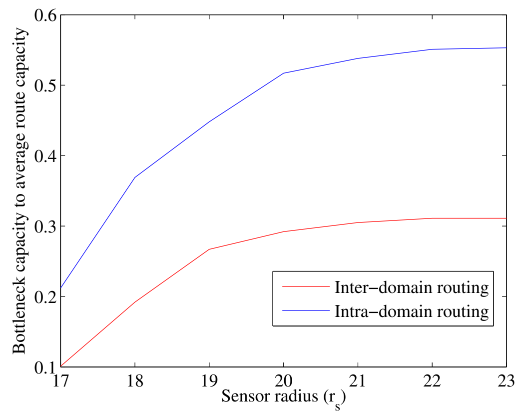
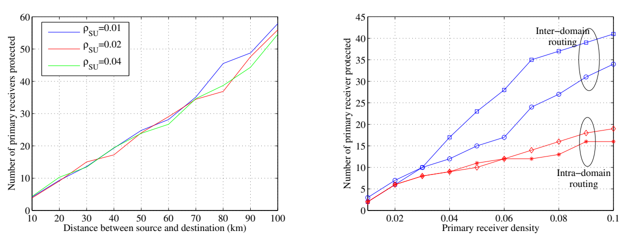


Fig. 7. Bottleneck capacity with sensor radius.



(a) (b)

Fig. 8. Primary receivers protected with (a) distance between the source and destination, (b) primary receiver density.

along the route. However, we observe that such nature is independent of the density of secondary nodes in the network. Fig. 8(b) shows that the number of primary receivers protected is increasing with increasing primary receivers in the network for four different source-destination pairs, with a couple of cases each for inter-domain and inter-domain routing. As expected, the number of primary receivers protected is higher for inter-domain routing as more hops means more receivers being interfered for no power control.

VI. Conclusions

In this paper, we discussed the challenges of routing in cognitive radio networks over traditional ad-hoc networks. We proposed a multi-channel multi-hop routing technique for data packet routing among secondary nodes with a aid of a spectrum map created by few sensors; a scheme that guarantees primary receiver protection. A *selective flooding* technique is also devised to spread the route requests in the network without causing network wide flooding overhead. We analyzed the connectivity condition among secondary nodes using a mapped graph and found the conditions for a minimum spanning tree to exist in such graph. Simulation results showed that the proposed routing scheme maximizes route capacity achieved and guarantees primary receiver protection.

References

[1] Y. Li, T. T. Quang, Y. Kawahara, T. Asami, and M. Kusunoki, “Building a spectrum map for future cognitive radio technology,” in *Proceedings of the 2009 ACM workshop on Cognitive radio networks*, ser. CoRoNet.

[2] T. Harrold, R. Cepeda, and M. Beach, “Long-term measurements of spectrum occupancy characteristics,” in *New Frontiers in Dynamic Spectrum Access Networks, DySPAN. IEEE Symposium on*, May 2011, pp. 83 –89.

[3] S. Debroy, S. Bhattacharjee, and M. Chatterjee, “Performance based channel allocation in ieee 802.22 networks,” in *Personal Indoor and Mobile Radio Communications, PIMRC. IEEE International Symposium on*, Sept. 2011, pp. 619–623.

[4] `https://www.google.com/get/spectrumdatabase/`.

[5] C. Xin, B. Xie, and C.-C. Shen, “A novel layered graph model for topology formation and routing in dynamic spectrum access networks,” in *New Frontiers in Dynamic Spectrum Access Networks, DySPAN. First IEEE International Symposium on*, Nov. 2005, pp. 308–317.

[6] Y. T. Hou, Y. Shi, and H. D. Sherali, “Spectrum sharing for multi-hop networking with cognitive radios,” *Selected Areas in Communications, IEEE Journal on*, vol. 26, no. 1, pp. 146–155, Jan. 2008.

[7] H.-P. Shiang and M. Van der Schaar, “Distributed resource management in multihop cognitive radio networks for delay-sensitive transmission,” *Vehicular Technology, IEEE Transactions on*, vol. 58, no. 2, pp. 941–953, Feb. 2009.

[8] K. R. Chowdhury and M. D. Felice, “Search: A routing protocol for mobile cognitive radio ad-hoc networks,” *Computer Communications Journal (ELSEVIER)*, vol. 32, no. 18, pp. 1983–1997, Dec. 2009.

[9] “Modelling and simulation of rayleigh fading, path loss, and shadowing fading for wireless mobile networks,” *Simulation Modelling Practice and Theory*, vol. 19, no. 2, pp. 626 – 637, 2011.

[10] R. Shrock and F. Y. Wu, “Spanning trees on graphs and lattices in d dimensions,” *Journal of Physics A: Mathematical and General*, vol. 33, no. 21, p. 3881.

[11] F. Y. Wu, “Number of spanning trees on a lattice,” *Journal of Physics A: Mathematical and General*, vol. 10, no. 6, p. L113.

[12] J. Riihijarvi and P. Mahonen, “Exploiting spatial statistics of primary and secondary users towards improved cognitive radio networks,” in *Cognitive Radio Oriented Wireless Networks and Communications, CrownCom. 3rd International Conference on*, May 2008, pp. 1 –7.

[13] “FCC adopts rule for unlicensed use of television white spaces,” `http://www.fcc.gov/`.